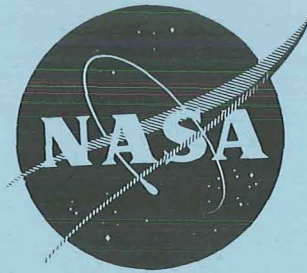


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DROPLET VAPORIZATION WITH  
LIQUID HEAT CONDUCTION

by

V. D. Agosta and S. S. Hammer

PROPULSION SCIENCES, INC.

prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

NASA Lewis Research Center  
Contract NAS 3-12030  
Richard J. Priem, Project Manager

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FINAL REPORT

DROPLET VAPORIZATION WITH  
LIQUID HEAT CONDUCTION

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

February 10, 1970

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## ABSTRACT

An analytical solution for droplet evaporation is presented. The governing differential equation includes finite liquid thermal conductivity within the droplet interior. The boundary conditions are a heat balance at the droplet surface, and finite temperatures at the droplet center. The effect of initial droplet size and temperature, droplet thermal conductivity and environmental gas pressure on temperature and vaporization rate histories are investigated. It is found that for the larger droplets, a dynamic equilibrium state is approached; i.e., the droplet surface regression rate parameter and the surface temperature approach common values.

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# DROPLET VAPORIZATION WITH LIQUID HEAT CONDUCTION

By V. D. Agosta and S. S. Hammer

## SUMMARY

The scope of this study is to explore the effect of liquid thermal conductivity on the behavior of an evaporating droplet. An analytical solution of the governing differential equation for heat conduction within the droplet interior is presented. The boundary conditions are finite temperature at the droplet center and a heat balance at the droplet surface. The effect of initial droplet size and temperature, droplet thermal conductivity and environmental gas pressure on temperature and evaporation rate histories are investigated.

It is found that for larger droplets, a common dynamic equilibrium state asymptotically is approached. The effect of thermal conductivity is to vary the time required for the droplet surface temperature to reach the asymptotic value. The time to vaporize ten percent of the initial droplet mass was not influenced by the thermal conductivity. The droplet equilibrium surface regression rate and temperature increases with environmental gas pressure. The droplet surface temperature reaches the critical value at an environmental gas pressure that is dependent on the volatility of the fluid. Finally, for droplets equal or greater than 100 microns diameter and with realistic thermal conductivities, the temperature gradients attained at  $10^{-4}$  sec. are contained in the outer ten percent of the droplet radius.

## INTRODUCTION

The burning rate for small fuel droplets is usually based on the quasi-steady envelope flame model (Refs. 1-4). Essentially, this model assumes that heat is transferred back from the flame envelope such that vaporization is sustained, and that the burning rate is equal to the flow of fuel from the interior of a droplet through a sphere of the same size and located in the same environment. It is further assumed that either the temperature at the droplet surface is near the boiling point or is uniform throughout the droplet. The quasi-steady droplet evaporation model appears to break down near the propellant critical point (Ref. 5).

The primary objective of the analysis presented herein is to determine the behavior of vaporizing droplets in which heat conduction in the liquid droplet is included in the model. This is accomplished by analyzing the droplet interior and employing the environment as boundary conditions. Since the droplet is the mass source, then it is argued that its behavior must be analyzed subject to the constraints of its environment. An additional bonus that becomes apparent is that the model presented herein can serve as one for the evaporation behavior of droplets near the droplet critical conditions.

It is our pleasure to acknowledge Dr. Gino Moretti, who developed the computer program for this study.

## THEORY

Initially, a droplet with uniform temperature is introduced into a hot atmosphere which is essentially at the adiabatic flame temperature. Droplet heating and vaporization occur simultaneously from the initial conditions and approach asymptotically a dynamic equilibrium state. The droplet vapors diffuse, mix and react chemically with the hot gases. Thus the physical structure of the model consists of a spherical droplet surrounded by a film of reacting vapor-gas mixture. The thickness of the film is defined as that distance where the reaction is in equilibrium for the local or given O/F ratio. Heat transfer,  $q_T$ , is assumed to occur backward toward the droplet which goes: (1) toward heating up the diffusing vapors,  $q_{sh}$ , (2) supplying the heat of vaporization,  $q_\lambda$ , and (3) heating up the droplet,  $q_\ell$ , Fig. 1.

The governing differential equation for heat conduction in a spherical liquid droplet is given by

$$T_t = \alpha(T_{rr} + \frac{2}{r} T_r) \quad 0 \leq r \leq r_s \quad (1)$$

$$t \geq 0$$

where

$T$  is the temperature,  
 $r$  radius,  $r_s$  surface radius,  
 $t$  time, and  
 $\alpha$  liquid thermal diffusivity.

The subscripts  $r$  and  $t$  represent differentiation with respect to

radius and time, respectively. For small droplets the assumption of spherical geometry is usually accepted. It is recognized that where drag exists, droplets deform; however, in this analysis the deformation is neglected. The initial condition assumes that the droplet is at uniform temperature  $T_o$ ,

$$T(r,0) = T_o \quad 0 \leq r \leq r_s \quad (2)$$

$$t = 0$$

The boundary conditions are

$$T(0,t) \text{ is finite} \quad r = 0 \quad (3)$$

$$t \geq 0$$

and

$$h(T_f - T_s) = \kappa T_r + \frac{\dot{w}}{A_d} [\lambda + (\overline{c_p})_v (T_f - T_s)] \quad (4)$$

$$r = r_s$$

$$0 \leq t \leq \infty$$

where

|                      |  |
|----------------------|--|
| $h$                  | is the heat transfer coefficient,      |
| $T_f$                | ambient (adiabatic flame) temperature, |
| $T_s$                | droplet surface temperature,           |
| $\kappa$             | thermal conductivity of droplet,       |
| $\lambda$            | heat of vaporization,                  |
| $(\overline{c_p})_v$ | vapor specific heat,                   |
| $\dot{w}$            | mass evaporation rate,                 |
| $A_d$                | droplet surface area.                  |

It is assumed that the film thickness surrounding the droplet is small compared to the droplet diameter.

The problem is still not completely defined because the mass evaporation rate is unknown. Thus, a constitutive equation is required and is

$$\rho_d \dot{r}_s = \frac{\dot{w}}{A_d} = K_g p \ln \frac{p}{p - p_v}, \quad r = r_s \quad (5)$$

where

$\dot{r}_s$  is the droplet surface regression rate,  
 $\rho_l$  the liquid density,  
 $K_l$  mass transfer coefficient,  
 $p^g$  total gas pressure,  
 $p_v$  vapor pressure corresponding to the droplet temperature surface.

It is assumed that the Clausius-Clapeyron equation relates the droplet surface temperature to the vapor pressure. A general relationship for this purpose is

$$\ln p_v = a + \frac{b}{T} + cT \quad (6)$$

where  $a$ ,  $b$ , and  $c$  are constants to be determined from thermodynamic data. The above Eq. (6) is also used for interpolation purposes. The heat of vaporization is a function of the droplet surface temperature and is given by

$$\lambda \left[ 1 - \frac{T_s}{T_c} \right]^\gamma \quad (7)$$

where  $\lambda_0$  and  $\gamma$  are constants and  $T_c$  the critical temperature. The average vapor specific heat,  $(\overline{c_p})_v$ , is either given by suitable equations, e.g.,

$$(\overline{c_p})_v = \frac{\int_{T_s}^{T_f} (\overline{c_p})_v dT}{T_f - T_s} \quad (8)$$

or from tables. The heat and mass transfer coefficients  $h$  and  $K_g$ , respectively, are obtained from the Nusselt correlations,

$$\frac{2r_s h}{K_m} = 2 + 0.6(Pr)^{1/3} (Re)^{1/2} \quad (9)$$

and

$$\frac{2r_s R_o T K_g}{M_v D} = 2 + 0.6 (Sc)^{1/3} (Re)^{1/2} \quad (10)$$

where

- $K_m$  is the vapor-gas mixture thermal conductivity,
- $R_o$  universal gas constant,
- $M_v$  vapor molecular weight,
- $D$  molecular diffusion coefficient,
- $\bar{T}$  arithmetic mean temperature,  $\frac{T_f + T_s}{2}$ ,
- $Pr$  Prandtl number  $(c_p \mu / \kappa)_m$ ,
- $Sc$  Schmidt number  $(\mu / D \rho)_m$ ,
- $Re$  Reynolds number  $2r_s (u-v) (\frac{\rho}{\mu})_m$ ,
- $\mu$  vapor-gas mixture viscosity,
- $\rho$  vapor-gas mixture density,
- $u$  gas velocity,
- $v$  droplet velocity.

It is recognized that most of the thermodynamic properties are functions of droplet surface temperature. However, to include such functional relationships at this time is impractical from a computational point-of-view.

A Laplace transform procedure was used to obtain a solution for the temperature profile in the liquid droplet employing the additional assumption that

$$\dot{r}_s \Delta t \ll r_s \quad (11)$$

The solution is given below (see Appendix A for derivation):



$$\begin{aligned}
T = T_O - \left[ \frac{T_O (1 + Ar_s) + B}{Ar} \right] \cdot \left\{ \operatorname{erfc} \frac{1-R}{2\sqrt{\tau}} - \operatorname{erfc} \frac{1+R}{2\sqrt{\tau}} - \right. \\
\exp(Ar_s (1-R) + A^2 r_s^2 \tau) \cdot \operatorname{erfc} \left( \frac{1-R}{2\sqrt{\tau}} + Ar_s \sqrt{\tau} \right) + \\
\left. \exp(Ar_s (1+R) + A^2 r_s^2 \tau) \cdot \operatorname{erfc} \left( \frac{1+R}{2\sqrt{\tau}} + Ar_s \sqrt{\tau} \right) \right\} \quad (12)
\end{aligned}$$

where  $R = \frac{r}{r_s}$

$$\tau = \frac{\alpha t}{r_s^2}$$

$$A = \left[ \frac{h}{\kappa} - \frac{1}{r_s} - \frac{(\bar{c}_p)_v}{\kappa} K_g p \ln \frac{p}{p-p_v} \right]$$

and

$$B = r_s \left[ \left( \frac{\lambda}{\kappa} + \frac{(\bar{c}_p)_v T_f}{\kappa} \right) K_g p \ln \frac{p}{p-p_v} - \frac{h T_f}{\kappa} \right].$$

The analysis of droplet evaporation in a gas stream, as formulated, is developed into a computer program. Once the initial conditions are specified at time  $t=0$  (e.g., the initial droplet radius, the initial uniform droplet temperature and the ambient conditions), one obtains a formula for the surface temperature of the droplet at any successive time,  $t_1$ . A number of coefficients in that equation depend on mean values of physical parameters over the entire span  $t=0, t_1$  (physical properties used in this study are given in Appendix B). Therefore, the computation has to be performed stepwise, in order to provide averaged intermediate values of the physical parameters between  $t=0$  and  $t=t_1$  (see Appendix C for calculation procedure and computer program).

The program is as follows: Time is increased by steps. At each step, the surface temperature and the droplet radius are averaged from the initial values to the values pertinent to that step. The mean physical properties are computed, according to

the formulae given. Then the new surface temperature is computed from the Laplace transform solution and the rate of decrease of droplet radius is evaluated. This allows the new droplet radius at the next step to be determined. Since some of the yet unknown new values have to be used in making the averages, one has to start each step with a suitable guess on the surface temperature and then find the right value by a trial-and-error technique.

The program is written in such a way that different gases and different liquids can be considered, by changing some of the data cards.

A computer run is terminated when ten percent of the initial droplet mass is evaporated, or when the droplet surface temperature reaches the propellant critical temperature. The regression rates and times are normalized by using steady state droplet evaporation rates and lifetimes. It is assumed a priori that for various droplet diameters, the droplet surface temperatures approach a common value during evaporation (i.e., a wet-bulb temperature) (Ref. 6). Thus, considering the equation for mass transfer from the droplet surface, Eq. (5),

$$\frac{\dot{w}}{A_d} = \dot{r}_s \rho_l = K_g p \ln \frac{p}{p-p_v} \quad (5)$$

the chamber pressure,  $p$ , and the liquid density,  $\rho_l$ , are given constants. The propellant vapor pressure,  $p_v$ , is constant because of the aforementioned assumption. Therefore, Eq. (5) reduces to

$$\dot{r}_s = C_1 K_g \quad (13)$$

where  $C_1$  is determined from the constant terms in Eq. (5). Nusselt's relation for mass transfer, Eq. (10) is given as

$$\frac{2r_s R_o \bar{T} K_g}{M_v D} = 2 + 0.6(Sc)^{1/3} (Re)^{1/2} \quad (10)$$

where  $r_s$  is the droplet radius.

Assuming that for high speed flow ,

$$0.6(Sc)^{1/2}(Re)^{1/2} > 2 \quad (14)$$

gives

$$K_g \propto r_s^{-1/2} \quad (15)$$

Combining Eqs. (13) and (15) gives

$$\dot{r}_s r_s^{1/2} \approx \text{constant} \quad (16)$$

If the decrease in droplet radius during the initial stages of evaporation is neglected,

$$r_s \approx r_o \quad (17)$$

Thus,

$$\dot{r}_s r_o^{1/2} = \text{constant} , \quad (18)$$

where  $r_o$  is the initial droplet radius.

This variable is used as the normalized surface regression rate. Integrating Eq. (16) between the limits  $0 \leq r_s \leq r_o$  and  $0 \leq t \leq t_d$ , where  $t_d$  is the droplet lifetime, gives

$$t_d \propto r_o^{3/2} \quad (19)$$

Thus, the nondimensional time is defined as

$$\frac{t}{t_d} \equiv C_2 \frac{t}{r_o^{3/2}} . \quad (20)$$

## RESULTS

The results for droplet evaporation for four propellant combinations are given. The parameters varied are: the initial droplet radius, the initial droplet temperature, the thermal conductivity, and the chamber pressure. The reference case for all propellants is an initial droplet diameter of 100 microns, initial temperature of the normal boiling point, and the chamber pressure equal to that of the evaporating propellant critical pressure.

### A. Variation in Droplet Histories with Initial Droplet Radius

In Fig. 2, the droplet surface regression rate parameter  $\dot{r}_s \sqrt{r_o}$  is plotted against the nondimensional time for the following propellant combinations:

- Fig. 2a 50/50 aeroxine with NTO
- Fig. 2b NTO with 50/50 aeroxine
- Fig. 2c oxygen with gaseous hydrogen
- Fig. 2d n-heptane with gaseous oxygen.

The propellants are at their normal boiling points and the ambient pressure is that of the propellant critical pressure.

Several general statements can be made for all these propellant combinations: (1) The surface regression rate parameter curves for all the droplet sizes do not coalesce during the heating up period, indicating different histories; (2) As time increases, the surface regression rate parameters approach a common value for the larger droplets. The surface regression rates for the smaller droplets (e.g., 25 $\mu$  droplet in particular) continue to increase, indicating that "steady state" behavior is not approached; (3) It is noticed that the curves for the surface regression rate parameter cross, and that the value of the nondimensional time at which crossover occurs decreases as the propellant becomes more volatile. This result suggests that the heating up period decreases with propellant volatility; (4) As the propellant becomes more volatile, the asymptotic value of the burning rate parameter increases.

In Fig. 3, the droplet surface temperature history is plotted against the nondimensional time. It is observed that in all cases,

a common surface temperature is approached asymptotically for each propellant. The reduced surface temperature, i.e.,  $T_s/T_c$ , where  $T_c$  is the critical temperature of the droplet propellant, varies from 0.77 for n-heptane to 0.97 for nitrogen tetroxide. From these results, it is seen that the reduced surface temperature increases as the volatility of the propellant becomes greater.

Fig. 4 shows the droplet temperature distribution with radius at  $10^{-4}$  sec. We observe two trends, namely: (1) for each propellant and for all droplet sizes, the surface temperatures approach a common value; and (2) steep positive temperature gradients occur in approximately the outer ten percent of the droplet radius. In one case, i.e., the 25 micron diameter droplet of n-heptane, the temperature gradient is broadened and extends to 30% of the droplet radius. However, if instead of plotting the droplet temperature distribution at the end of  $10^{-4}$  sec., the nondimensional time at the crossover point is used, then for the case of the 25 micron droplet of n-heptane, the surface temperature gradient is contained in the outer ten percent of the droplet radius.

#### B. Variation in Droplet Histories with Initial Droplet Temperature

In the following series of computer tests, the initial droplet temperature was varied. In these cases the initial droplet diameter was assumed to be 100 microns, and the gas pressure that of the droplet propellant critical pressure. Three runs were made with the droplet initially at: (1) the propellant normal boiling point, (2) the temperature  $[T_{nbp} + 5/6 T_c]/2$ , and (3) the temperature  $5/6 T_c$ . The results of these tests are plotted in Figs. 5, 6 and 7. In Fig. 5, the surface regression rate parameter is plotted against the reduced time. It is seen that with 50/50 aerazine and n-heptane, the surface regression rate parameter approaches an asymptotic value independent of initial propellant temperature. With NTO and oxygen, the curves do not appear to cross within the real time of  $10^{-4}$  sec. Certainly the crossover point, if any, has translated to larger values of time. Similar behavior is observed for the surface temperatures versus time, Fig. 6; namely, it is seen that a common value of surface temperature is approached asymptotically for each propellant.

In Fig. 7, the temperature distribution within the droplet is shown for the time  $10^{-4}$  sec. Again it is seen that the surface temperatures approach a common value for each propellant, and that the temperature gradient is located in the outer ten percent of the droplet radius. One significant departure is observed for n-heptane with an initial temperature of  $810^{\circ}\text{R}$ . The temperature gradient at the surface is negative. In this case the initial droplet temperature was above the equilibrium surface temperature. The dynamic balance of heat and mass transfer, Eq. (4), requires that part of the droplet internal energy be used to supply the heat of vaporization.

### C. Variation in Droplet Histories with Droplet Thermal Conductivity

In the following computer test runs, the effect of liquid thermal conductivity on the droplet evaporation behavior and temperature distribution was considered. The results of this study are plotted in Figs. 8, 9 and 10. In all these tests, the data extends to where ten percent of the droplet mass has evaporated. The liquid thermal conductivity was varied between  $10^{-5}$  and  $10^{-1}$  BTU ft.<sup>-1</sup>sec.<sup>-1</sup> $^{\circ}\text{R}^{-1}$ . Values lower than  $10^{-5}$  or greater than  $10^{-1}$  caused computational procedure errors due to the form of the equations.

In Fig. 8, the surface regression rate parameter is plotted versus the nondimensional time. For 50/50 Aerozine and n-heptane, steady state evaporation is not approached for the cases where the larger thermal conductivities are used. This observation is based on the fact that the latter curves do not level off and approach common asymptotic values of the surface regression rate by the time ten percent of the mass is vaporized. The more volatile propellants are less sensitive to the value of thermal conductivity and level off toward common values of regression rate, thus attaining equilibrium values sooner. For all cases it is noticed that for smaller values of thermal conductivity, the surface regression rate parameter increases faster. The same general remarks can be made based on the surface temperature data given in Fig. 9. In Fig. 10, the droplet temperature distribution is shown. The results agree with intuition in that the greater the liquid thermal conductivity, the more uniform is the droplet temperature.



In passing it is noted that the time required for a 100 micron droplet to reach the ninety percent mass point was about the same, i.e.,  $5 \times 10^{-5} \leq t_{90\%} \leq 10^{-4}$  sec., regardless of the thermal conductivity. This behavior is due to the fact that only a small fraction of heat transfer,  $q_T$ , goes toward heating up the droplet,  $q_d$ . However, the behavior of these heated droplets in a time dependent environment could be vastly different.

#### D. Variation in Droplet Histories with Environment Gas Pressure

In this series of computer tests, the environmental gas pressure was increased from the propellant critical pressure to that where the droplet surface temperature reached the propellant critical temperature. In the data no evidence is seen of an approach to an asymptotic value of surface temperature. This behavior appears to be in agreement with that observed by Faeth (Ref. 7).

In Fig. 11, the droplet surface temperatures are plotted versus the reduced gas pressure,  $p_r$ , for 100 micron droplets initially at normal boiling point temperature. As the pressure increases, the surface temperature increases. The values plotted represent the droplet surface temperature at a time when ten percent of the droplet mass has evaporated. For this ninety percent mass point, the reduced pressure at which the critical temperature is reached at the surface is:

|                |                |
|----------------|----------------|
| n-heptane      | : $p_r = 8.3$  |
| 50/50 Aerozine | : $p_r = 4.1$  |
| NTO            | : $p_r = 1.37$ |
| O <sub>2</sub> | : $p_r = 1.26$ |

It is observed that as the propellant volatility increases, the reduced pressure decreases.

In Fig. 12, the surface regression rate is shown as a function of chamber pressure. It is seen that the surface regression rate increases as the chamber pressure increases. The trend, therefore, is for the droplet lifetime to decrease as the chamber pressure increases. This is in agreement with the work of Faeth (Ref. 7).

## SUMMARY OF RESULTS

The evaporative behavior of a liquid propellant droplet has been determined from a solution to a system of equations which include heat transfer within the droplet. In general, for droplets larger than 100 microns in diameter, the droplet heats up within  $10^{-4}$  sec and approaches a steady state of evaporation. Smaller droplets, e.g., 25 micron diameter droplets, do not approach a steady state behavior. The evaporative behavior of droplets with diameter larger than 100 microns is correlated by the surface regression rate parameter,  $\dot{r}_s \sqrt{r_0}$  and the nondimensional time,  $t/t_b$  after steady state have been achieved.

Analytical results obtained from the solution are:

(1) Regardless of the droplet initial temperature or radius, a common value of the droplet surface temperature is asymptotically approached. Therefore, where the droplet is heat transfer limited, or where the droplet initial temperature is greater than the steady state surface temperature, the droplet internal energy can contribute to the heat of vaporization.

(2) The effect of thermal conductivity is to vary the time required for the droplet surface temperature to reach the asymptotic value. The time required to evaporate ten percent of the droplet mass was not influenced by the value of the liquid thermal conductivity.

(3) The droplet equilibrium surface regression rate and temperature increases with environmental gas pressure. The droplet surface temperature reaches the critical value at an environmental gas pressure that is dependent on the volatility of the fluid.

(4) For droplets equal to or greater than 100 microns diameter and with realistic thermal conductivities, the temperature gradients attained at  $10^{-4}$  sec are contained in the outer ten percent of the droplet radius.

## APPENDIX A

### ANALYTIC SOLUTION FOR TEMPERATURE DISTRIBUTION

A short time solution for the temperature distribution in a liquid sphere where evaporation occurs at the surface is obtained by utilizing Laplace transformations. Eq. (1) can be transformed to a more convenient form by the substitution

$$U = Tr \quad (A-1)$$

which gives

$$U_t = \alpha U_{rr} \quad \begin{matrix} 0 \leq r \leq r_s \\ t \geq 0 \end{matrix} \quad (A-2)$$

The initial condition expressed in Eq. (2) is

$$U(r,0) = rT_o \quad \begin{matrix} 0 \leq r \leq r_s \\ t=0 \end{matrix} \quad (A-3)$$

and the boundary conditions corresponding to Eqs. (3a) and (3b) are

$$U(0,t) = 0 \quad \begin{matrix} r=0 \\ t \geq 0 \end{matrix} \quad (A-4a)$$

$$U_r = U \left[ \frac{1}{r_s} - \frac{h}{\kappa} + \frac{(\overline{c_p})_v}{\kappa} \dot{r}_{s\rho_\ell} \right] - \dot{r}_{s\rho_\ell} \frac{r_s}{\kappa} \left[ \lambda + (\overline{c_p})_v T_f \right] + \frac{r_s h}{\kappa} T_f \quad \begin{matrix} r=r_s \\ t \geq 0 \end{matrix} \quad (A-4b)$$

It is assumed that  $\dot{r}_s \Delta t \ll r_s$ , and that  $\lambda$  and  $p_v$  are constants over the time interval  $\Delta t$ .

The Laplace transform is taken of Eq. (A-2),

$$\int_0^\infty e^{-pt} U_{rr} dt - \frac{1}{\alpha} \int_0^\infty e^{-pt} U_t dt = 0 \quad (A-5a)$$

Expanding and including the initial condition of Eq. (A-3) gives

$$\bar{U}_{rr} - q^2 \bar{U} = - \frac{r}{\alpha} T_o \quad (A-5b)$$

$$\text{where } q^2 \equiv \frac{p}{\alpha} . \quad (A-5c)$$

The boundary conditions of Eqs. (A-4a) and (A-4b) becomes

$$\begin{aligned} \bar{U}(0, t) &= 0 & r=0 \\ & & t \geq 0 \end{aligned} \quad (A-6a)$$

$$\begin{aligned} \bar{U}_r + A\bar{U} + \frac{B}{p} &= 0 & r=r_s \\ & & t \geq 0 \end{aligned} \quad (A-6b)$$

where

$$A \equiv \left[ \frac{h}{\kappa} - \frac{1}{r_s} - \frac{(\bar{c}_p)_v K_g p}{\kappa} \ln \frac{p}{p-p_v} \right] \quad (A-7a)$$

and

$$B \equiv r_s \left[ \left( \frac{\lambda}{\kappa} + \frac{(\bar{c}_p)_v T_f}{\kappa} \right) K_g p \ln \frac{p}{p-p_v} - \frac{h T_f}{\kappa} \right] \quad (A-7b)$$

The general solution to Eq. (A-5b) is

$$\bar{U} = C_1 e^{qr} + C_2 e^{-qr} + \frac{T_o r}{q^2 \alpha} \quad (A-8)$$

Substitution into boundary condition (A-6a) gives

$$C_2 = - C_1 \quad (A-9a)$$

and into Eq. (A-6b) gives

$$C_1 = - \frac{\frac{T_o}{q^2 \alpha} [1 + A r_s] + \frac{B}{p}}{(A+q) e^{q r_s} - (B-q) e^{-q r_s}} \quad (A-9b)$$

Thus the solution becomes

$$\bar{U} = \frac{T_o r}{p} - \left[ \frac{T_o (Ar_s + 1) + B}{p} \right] \cdot \sum_{n=0}^{\infty} (-1)^n \frac{(q-A)^n}{(q+B)^{n+1}} \left[ e^{-q(2n+1)(r_s-r)} - e^{-q(2n+1)(r_s+r)} \right] \quad (A-10)$$

For a solution valid over a short interval of time, the series can be truncated after the  $n=0$  term. This was verified by extending the series and testing for convergence. The inverse transform of Eq. (A-10) is

$$T = T_o - \left[ \frac{T_o (1+Ar_s) + B}{Ar} \right] \cdot \left\{ \operatorname{erfc} \frac{1-R}{2\sqrt{\tau}} - \operatorname{erfc} \frac{1+R}{2\sqrt{\tau}} - \exp(Ar_s(1-R) + A^2 r_s^2 \tau) \cdot \operatorname{erfc} \left( \frac{1-R}{2\sqrt{\tau}} + Ar_s \sqrt{\tau} \right) + \exp(Ar_s(1+R) + A^2 r_s^2 \tau) \cdot \operatorname{erfc} \left( \frac{1+R}{2\sqrt{\tau}} + Ar_s \sqrt{\tau} \right) \right\} \quad (A-11)$$

A qualitative discussion of the solution is now given. If we refer to Eq. (A-11), we find that it is made up of two parts. The first is,  $T_o$ , and it is recognized as the droplet initial temperature. The second is the product of

$$\left[ \frac{T_o (1+Ar_s) + B}{Ar} \right] \cdot F(r, t)$$

and is the droplet heatup and vaporization due to the backward heat transfer from the hot environment, i.e., for the case where the environment is considered steady in temperature. Therefore, the two parts make up the solution to the heat and mass transfer

to a burning droplet in a constant temperature environment. From summary slide rule calculations it is found that for hydrazine-type and hydrocarbon-type fuels, droplet heatup times are about  $10^{-4}$  sec. In addition, steep temperature gradients occur at the droplet surface, the interior apparently not affected in the absence of radiation and convection. Thus the transient behavior of such a droplet is different from one with infinite thermal conductivity (i.e., a constant temperature droplet) due to decreased thermal inertia.

## APPENDIX B

### THERMODYNAMIC CONSTITUTIVE EQUATIONS

The thermodynamic properties for the propellant combinations employed in this study are given below. Where mixtures are considered, then Gibbs-Dalton's Law is used:

$$\text{pressure , } p = \sum_i p_i \quad (\text{B-1})$$

$$\text{enthalpy , } h = \sum_i y_i h_i \quad (\text{B-2})$$

$$\text{entropy , } s = \sum_i y_i s_i \quad (\text{B-3})$$

where  $y_i$  is the mass fraction, and for transport properties,

$$\theta = \sum_i \frac{x_i \theta_i}{\sum_j x_j \varphi_{ij}} \quad (\text{B-4})$$



where

$$\varphi_{ij} = \frac{1}{\sqrt{8}} \left(1 + \frac{M_i}{M_j}\right)^{-\frac{1}{2}} \left[1 + \left(\frac{\theta_i}{\theta_j}\right)^{\frac{1}{2}} \left(\frac{M_i}{M_j}\right)^{\frac{1}{4}}\right]^2 \quad (\text{B-5})$$

and  $\theta$  is the transport property,  
 $M$  the molecular weight,  
 $x$  the mole fraction.

For the propellant combinations, 50/50 Aerozine with nitrogen tetroxide, oxygen with gaseous hydrogen, and nitrogen tetroxide with 50/50 Aerozine, the following constitutive equations are used:

Diffusion coefficient equation,

$$D_r = \frac{2.33 \times 10^{-6} (P_{CA} P_{CB})^{1/3} (T_{CA} T_{CB})^{0.416}}{\sqrt{\frac{M_A M_B}{M_A + M_B}}} \cdot \frac{1}{p} \left(\frac{\bar{T}}{\sqrt{T_{CA} T_{CB}}}\right)^{1.823} \quad (\text{B-6})$$

Vapor pressure - Temperature relationship,

$$p_v = \exp\left(a + \frac{b}{T} + cT\right) \quad (\text{B-7})$$

Heat of vaporization relationship,

$$\lambda = \lambda_o \left[1 + \frac{T_s}{T_c}\right]^\gamma \quad (\text{B-8})$$

where  $p_c$  is the critical pressure;  $T_c$  the critical temperature;

$\bar{T}$  the arithmetic mean temperature;  $p_v$  the vapor pressure;  $a, b, c$  constants;  $\lambda, \lambda_o$  heats of vaporization; and  $T_s$  the droplet surface temperature. The subscript A refers to the propellant under investigation, the subscript B to the remaining mixture under investigation. Thermodynamic equilibrium is assumed for the mixture B in the film surrounding the droplet at a mean temperature  $T = [T_f + T_s]/2$ . Dissociation energies are included in the determination of the thermal and transport properties. Average values are determined for the following properties and assumed constant: liquid thermal conductivity, liquid density, liquid specific heat, vapor molecular weight, vapor specific heat, gas mixture apparent molecular weight, gas mixture viscosity, gas mixture thermal conductivity, gas mixture specific heat.

For the case of n-heptane with gaseous oxygen, the thermodynamic properties used are those given in Ref. 18. The thermodynamic properties used in this study are given below:

#### 50/50 Aerozine with NTO

|                      |  |
|----------------------|--|
| $\alpha$             | $1.805 \times 10^{-6} \text{ ft}^2 \text{ sec}^{-1}$                                     |
| $\rho$               | $56.5 \text{ lbm. ft}^{-3}$  |
| $P_{CA}$             | $2.44 \times 10^5 \text{ lbf ft}^{-2}$   |
| $P_{CB}$             | $2.37 \times 10^5 \text{ lbf ft}^{-2}$   |
| $T_{CA}$             | $1094^\circ \text{R}$  |
| $T_{CB}$             | $629^\circ \text{R}$   |
| $M_A$                | 45.0   |
| $M_b$                | 22.5   |
| $\mu$                | $4.30 \times 10^{-5} \text{ lbm ft}^{-1} \text{ sec}^{-1}$                               |
| Pr                   | 0.70306  |
| $\kappa_B$           | $3.54 \times 10^{-5} \text{ Btu ft}^{-1} \text{ sec}^{-1} \text{ } ^\circ \text{R}^{-1}$ |
| $(\overline{c_p})_v$ | $1.100 \text{ Btu lbm}^{-1} \text{ } ^\circ \text{R}^{-1}$                               |
| $\kappa$             | $4.20 \times 10^{-5} \text{ Btu ft}^{-1} \text{ sec}^{-1} \text{ } ^\circ \text{R}^{-1}$ |
| a                    | 17.167   |
| b                    | 6381.9   |

c 0.0010199  
 $\lambda_o$  538.0 Btu lbm<sup>-1</sup>

NTO with 50/50 Aerozine

$\alpha$   $1.0378 \times 10^{-6} \text{ ft}^2 \text{ sec}^{-1}$   
 $\rho$  89.34 lbm ft<sup>-3</sup>  
 $P_{CA}$   $2.06 \times 10^5 \text{ lbf ft}^{-2}$   
 $P_{CB}$   $2.37 \times 10^5 \text{ lbf ft}^{-2}$   
 $T_{CA}$  776<sup>o</sup>R  
 $T_{CB}$  629<sup>o</sup>R  
 $M_A$  46.0 (NO<sub>2</sub> diffuses in film B)  
 $M_B$  22.5  
 $\mu$   $4.30 \times 10^{-5} \text{ lbm ft}^{-1} \text{ sec}^{-1}$   
 $Pr$  0.70306  
 $K_B$   $3.54 \times 10^{-5} \text{ Btu ft}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{R}^{-1}$   
 $(c_p)_v$  2.900 Btu lbm<sup>-1</sup>°R<sup>-1</sup>  
 $n$   $2.16 \times 10^{-5} \text{ Btu ft}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{R}^{-1}$   
 $a$  17.844  
 $b$  6300.0  
 $c$  0.0032450  
 $\lambda_o$  268.0

Oxygen with gaseous Hydrogen

$\alpha$   $7.5677 \times 10^{-7} \text{ ft}^2 \text{ sec}^{-1}$   
 $\rho$  72.0 lbm ft<sup>-3</sup>  
 $P_{CA}$   $1.06 \times 10^5 \text{ lbf ft}^{-2}$   
 $P_{CB}$   $4.10 \times 10^5 \text{ lbf ft}^{-2}$   
 $T_{CA}$  278.16<sup>o</sup>R  
 $T_{CB}$  1160<sup>o</sup>R  
 $M_A$  32.0  
 $M_B$  18.0

|           |  |
|-----------|--|
| $\mu$     | $4.20 \times 10^{-5} \text{ lbm ft}^{-1} \text{ sec}^{-1}$                               |
| Pr        | 0.71213  |
| $K_B$     | $4.04 \times 10^{-5} \text{ Btu ft}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{R}^{-1}$  |
| $(c_p)_v$ | $2.210 \times 10^{-5} \text{ Btu ft}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{R}^{-1}$ |
| a         | 16.928   |
| b         | 1476.50  |
| c         | 0  |

### Heptane with Gaseous Oxygen

|           |   |
|-----------|---|
| $\alpha$  | $5.391 \times 10^{-7} \text{ ft}^2 \text{ sec}^{-1}$                                    |
| $\rho$    | $67.80 \text{ lbm ft}^{-3}$   |
| $P_{CA}$  | $5.67 \times 10^4 \text{ lbf ft}^{-2}$  |
| $T_{CA}$  | $972^\circ\text{R}$   |
| $M_A$     | 100.2   |
| $M_B$     | 31.0  |
| $\mu$     | $5.43 \times 10^{-5} \text{ lbm ft}^{-1} \text{ sec}^{-1}$                              |
| Pr        | 2.0499  |
| $K_B$     | $1.29 \times 10^{-5} \text{ Btu ft}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{R}^{-1}$ |
| $(c_p)_v$ | $1.085 \text{ Btu lbm}^{-1} \text{ } ^\circ\text{R}^{-1}$                               |
| $\kappa$  | $2.21 \times 10^{-5} \text{ Btu ft}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{R}^{-1}$ |

## APPENDIX C

### CALCULATION PROCEDURE

#### Method

The procedure used in calculating the droplet evaporation histories and temperature distributions is illustrated by the flow diagram. The droplet surface temperature is given by Eq. (A-11), which requires a knowledge of time averaged thermodynamic and transport properties which in turn are functions of the droplet surface temperature history. At each succeeding time step the

surface temperature is first assumed constant, thermodynamic and transport properties are calculated and the surface temperature at the end of the time step is determined from Eq. (A-11). The thermodynamic properties are reevaluated with the calculated surface temperature and Eq. (A-11) is solved. The procedure continues until the assumed surface temperature and calculated surface temperature differ by less than  $10^{-4}$ . The surface regression rate, mass evaporation rate, droplet radius, and droplet temperature distribution are then determined. The time is incremented and the procedure continues.

#### Flow Diagram

Step 1. Load into the machine the following boundary conditions, initial conditions, and computational parameters:

Card 1. (I5) KOUNTA - Number of test cases to be analyzed.  
Card 2. (18A4) TITLE - Title or description of test case.  
Card 3. (7I5) NRUN - Number of test case.  
MONTH - Month  
MDAY - Day  
MYEAR - Year  
KA - Maximum number of time steps to be calculated.  
JA - Frequency at which the temperature distribution is calculated and printed, i.e., if JA=10 the temperature distribution is calculated on every 10th step.  
NP - Controls frequency of printing droplet radius, regression rate, surface temperature, etc.  
NP=+1 Print the first 100 steps and every JA step thereafter.  
NP=-1 Print every step.  
Card 4. (8E10.4)  
P - Chamber pressure (lbf/ft<sup>2</sup>)  
TF - Flame temperature (°R)  
RS - Initial drop radius (ft)  
TO - Initial drop temperature (°R)  
U - Gas velocity (ft/sec)  
V - Drop velocity (ft/sec)  
DT - Time increment (sec.)  
TEND - Droplet lifetime at which calculations end (sec)

Card 5. (8E10.4)

CPL - Specific heat-liquid (Btu/lbm<sup>°R</sup>)  
RHOL - Liquid density (lbm/ft<sup>3</sup>)  
AKL - Liquid thermal conductivity (Btu/ft sec<sup>°R</sup>)  
CPV - Propellant vapor specific heat (Btu/lbm<sup>°R</sup>)  
EMV - Propellant vapor molecular weight  
VIS - Combustion gas viscosity (ft-sec/lbm)  
CPB - Combustion gas specific heat (Btu/lbm<sup>°R</sup>)  
AKB - Combustion gas thermal conductivity (Btu/ft-sec<sup>°R</sup>)

Card 6. (8E10.4)

PCA - Critical pressure-propellant (.bf/ft<sup>2</sup>)  
PCB - Critical pressure-combustion gases (lb/ft<sup>2</sup>)  
TCA - Critical temperature-propellant (°R)  
TCB - Critical temperature-combustion gases (°R)  
EMA - Molecular weight-propellant  
EMB - Molecular weight-combustion gases

Card 7. (4E10.4)

PVA, PVB, PVC, PVD - Values of constants A,B,C,D in  
equation for propellant vapor pressure

$$P_v(T) = \exp \left( A - \frac{B}{T-C} + DT \right)$$

Card 8. (4E10.4)

SLA, SLB, SLC, SN1 - The first three are values of  
constants A,B,C in equation for latent heat of  
vaporization. SN1 determines which of two  
general equation forms is used.

$$SN1 = 1 \quad \lambda(T) = A \left( 1 - \frac{T}{B} \right)^C$$

$$SN1 = 0 \quad \lambda(T) = A + BT - CT^2$$



Card 9. (4E10.4)

DVA, DVB, DVC, DN1 - The first three are values of constants A,B,C in equation for diffusivity. DN1 determines which of two general equation forms is used.

$$\text{DN1}=1 \quad D_v = \frac{2.33(10^{-6})(P_{CA} \cdot P_{CB})^{1/3}}{\sqrt{\frac{M_A \cdot M_B}{M_A + M_B}}} \frac{(T_{CA} \cdot T_{CB})^{.416}}{(\frac{\bar{T}}{\sqrt{T_{CA} \cdot T_B}})^{1.823}}$$

$$(\frac{\bar{T}}{\sqrt{T_{CA} \cdot T_B}})^{1.823}$$

$$\text{DN1}=0 \quad D_v = (A+BT+CT^2) (\frac{300}{P})$$

Step 2. Calculate the initial drop mass, droplet surface regression rate and evaporation rate based upon initial conditions.

Step 3. Increment time and assume droplet surface temperature equal to that calculated at last time increment.

Step 4-5-6. Calculate the following time averaged properties:  $r_s$ ,  $T_s$ ,  $T_{\text{film}}$ ,  $P_v$ . A time averaged property is determined summing the product of the property and time increment over all of the increments and dividing by the total elapsed time. The heat transfer coefficient, mass transfer coefficient and heat of vaporization are then evaluated using the time averaged properties. Equation (A-11) is then solved for surface temperature.

Step 7,8. The calculated value of surface temperature is compared with the assumed value. If the difference exceeds .0001 the last calculated surface temperature is assumed (Step 8) and steps 4,5,6 are repeated until convergence is obtained.

Step 9. The mass transfer coefficient is recalculated based upon the present values of surface temperature, film temperature, droplet radius and vapor pressure. The instantaneous regression rate

$$\dot{r}_s = K_g \frac{P}{\rho_L} \ln\left(1 - \frac{P_v(T_s)}{P}\right)$$

and evaporation rate

$$\dot{m} = \rho_L 4\pi r_s^2 \dot{r}_s$$

are calculated.

Step 10. The temperature distribution ( $T$  as a function of  $r/r_s$ ) in the drop is calculated by solving Eq. (A-11) at  $R=r/r_s=.1,.2,.3,\dots,1.0$  using time averaged properties.

Step 11. The calculation results for the last time increment are printed. These include: temperature distribution, step number, time, drop radius, radius regression rate, mass evaporation rate, surface temperature, mass transfer coefficient, heat transfer coefficient, Schmidt number, Reynolds number, diffusivity and the ratio of the present drop mass to the initial mass.

Step 12. Several tests are made to determine if calculations should continue, i.e., should control transfer to step 3 for an additional time increment. If the number of steps already completed is greater than some input number ( $KA$ ), or if the time is greater than some pre-determined quantity ( $TEND$ ), or if the ratio of present mass to initial mass is less than .9, the calculations on the present test case will terminate and the next set of boundary conditions will be read.

In addition to the above described tests, there are additional checks built into the program. If the solution for surface temperature requires more than 20 iterations, the calculations terminate, a diagnostic message is printed and the next case is read. The calculations will also terminate with an appropriate

message if:

- a) drop vapor pressure is greater than chamber pressure,
- b) drop temperature is greater than critical temperature,
- c) drop radius becomes negative.

The program contains a built-in variation in time increment. The input value of DT (time increment) is used until the time equals 100DT. At that time the time increment is increased by a factor of 10. The time increment is increased whenever the total elapsed time equals 100 times the present time increment.

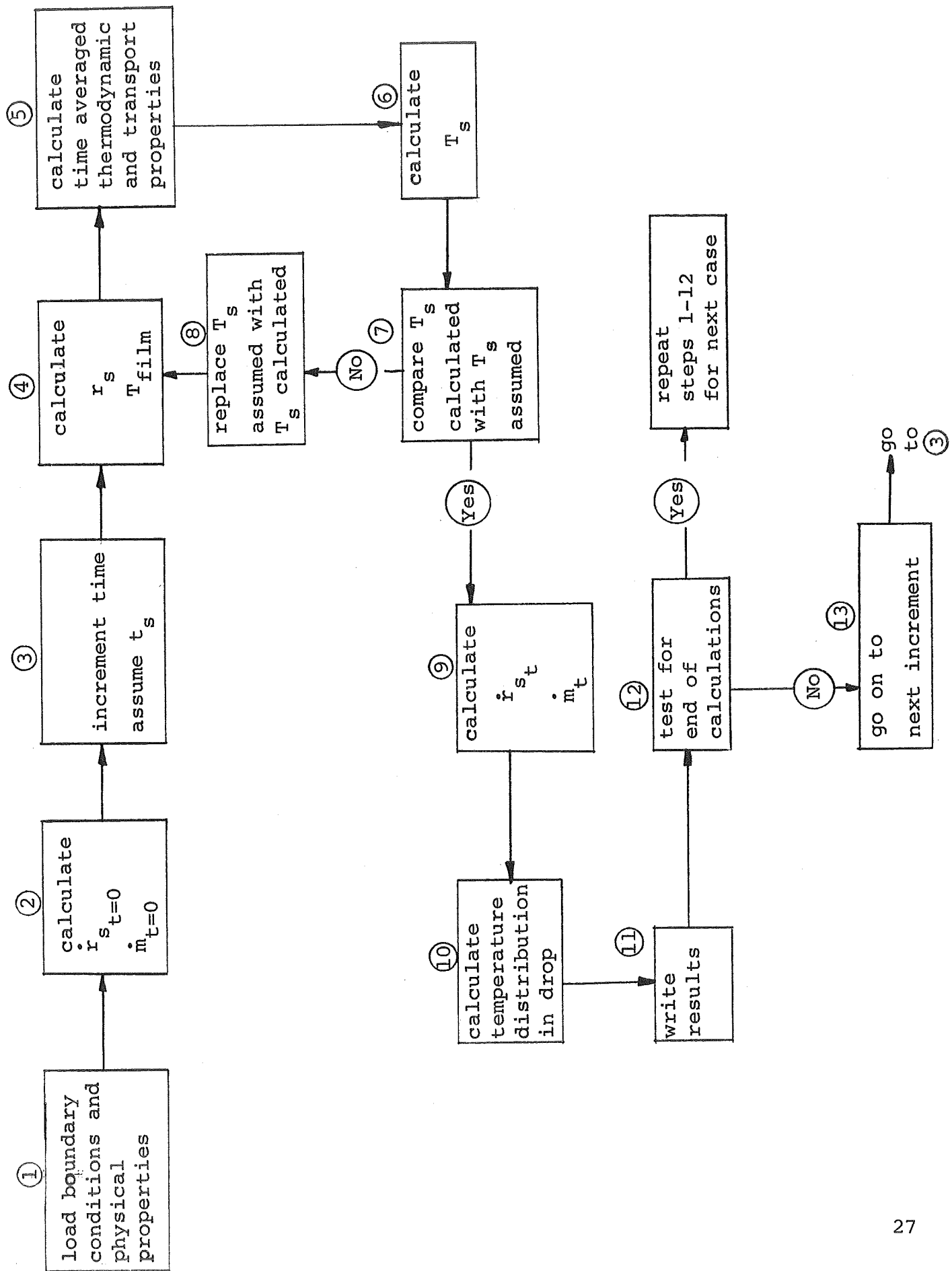


FIG. 13. FLOW DIAGRAM FOR COMPUTER PROGRAM

PROGRAM DROPS(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

C PROGRAM DROPS, MAIN

C  
000003 DIMENSION TS1(2),ERR(2),TITLE(18)

C  
000003 100 FORMAT(14I5)

000003 101 FORMAT(8E10.4)

000003 102 FORMAT(1H0,11H RUN NUMBER I5,5X,2H0N I3,1H/I2,1H/I2//11H STOP AFTER I  
16,18H STEPS OR AT TIME= E12.4/13H OUTPUT EVERY I6,6H STEPS//14H GAS  
2 PRESSURE= E13.5,7H LB/FT2/17H GAS TEMPERATURE= E13.5,8H RANKINE  
3/24H DROPLET INITIAL RADIUS= E13.5,3H FT/21H DROPLET TEMPERATURE=  
4 E13.5,8H RANKINE/14H GAS VELOCITY= E13.5,7H FT/SEC/18H DROPLET V  
5 VELOCITY= E13.5,7H FT/SEC/15H TIME STEPSIZE= E13.5,4H SEC)

000003 103 FORMAT(9H0 ALPHA,7X,4HRHOL,6X,12HGAS CONSTANT,3X,3HPCA,9X,3HPCB,  
19X,3HTCA,9X,3HTCB/ 7E12.4//4X,2HMA,10X,2HMB,10X,2HMV,11X,1HA,11X,  
21HB,9X,3HVIS,9X,2HPR/ 7E12.4//4X,2HKB,8X,5H(CP)V,7X,5HKAPPA/ 3E12  
2.4)

000003 104 FORMAT(11H0PV(T)=EXP(F7.3,1H-F8.3,4H/(T-F7.3,2H)+F10.7,3H\*T))

000003 105 FORMAT(16H0ITERATION FAILS)

000003 106 FORMAT(I7, 7E13.5, 3E9.2, F6.3)

000003 107 FORMAT(1H0,5X,1HK,5X,4HTIME,10X,2HRS,10X,5HRSDOT,9X,4HWDOT,10X,2HT  
1S,10X,2HKG,11X,1HH,8X,3HSCH,8X,3HREY,8X,2HDV,3X,4HM/MI/1H0)

000003 108 FORMAT(F35.1, E15.5)

000003 109 FORMAT(1H0///17X,47HTEMPERATURE DISTRIBUTION IN THE DROPLET AT STE  
1PI6//30X,4HR/RS,12X,1HT)

000003 110 FORMAT(1H1,30X,25HPROPULSION SCIENCES, INC./36X,12HP.0. BOX 814/  
133X,18HMELVILLE, NEW YORK//22X,45HWORK PERFORMED UNDER NASA CONTRA  
2CT NAS3-12,30 )

000003 111 FORMAT(1H0///17X,52HDROP VAPOR PRESSURE IS GREATER THAN CHAMBER PR  
1ESSURE )

000003 112 FORMAT(18A4)

000003 113 FORMAT(1H0//20X,18A4)

000003 114 FORMAT(11H0LAMBDA(T)=F7.2,1H+F8.4,3H\*T-F10.7,5H\*T\*\*2)

000003 115 FORMAT(11H0LAMBDA(T)=F7.2,6H(1.-T/F7.2,3H)\*\*F7.4)

000003 116 FORMAT(83H0D(P,T)=(PCA\*PCB)\*\*.33/SQRT(MA\*MB/(MA+MB))\*(TCA\*TCB)\*\*5/  
112/P\*A\*(T/SQRT(TCA\*TCB))\*\*B)

000003 117 FORMAT(9H0D(P,T)=(E13.5,1H+E13.5,3H\*T+E13.5,13H\*T\*\*2)\*300./P)

000003 118 FORMAT(34H0DROP RADIUS IS NEGATIVE AT TIME =E13.5)

000003 119 FORMAT(55H0DROP TEMPERATURE CAUSING VAPOR PRESSURE FCN TO BLOW UP)

000003 120 FORMAT(47H0MASS REMAINING IS LESS THAN .9 OF INITIAL MASS)

C  
000003 PV(X)=EXP(PVA-PVB/(X-PVC)+PVD\*X)

000017 D1(X,Y)=(PCA\*PCB)\*\*.333333/SQRT(EMA\*EMB/(EMA+EMB))\*(TCA\*TCB)\*\*.416  
1667/X\*\*AA\*(Y/SQRT(TCA\*TCB))\*\*BB

000056 D2(X,Y)=(DVA+DVB\*Y+DVC\*Y\*\*2)\*300./X

000072 SL1(X)=SLA\*(1.-X/SLB)\*\*SLC

000103 SL2(X)=SLA+SLB\*X-SLC\*X\*\*2.

000115 D(X,Y)=DN1\*D1(X,Y)+ABS(DN1-1.)\*D2(X,Y)

000135 RR=1545.

000136 AA=2.33E-6

000140 BB=1.823

C  
000141 KOUNT=0

000143 READ(5,100)KOUNTA

000150 1 KOUNT=KOUNT+1

000152 IF(KOUNT.GT.KOUNTA)CALL EXIT

000155 READ(5,112)TITLE

```

000163      READ(5,100)NRUN,MONTH,MDAY,MYEAR,KA,JA,NP
000205      READ(5,101)P,TF,RS,T0,U,V,DT,TEND,CPL,RHOL,AK,CPV,EMV,VIS,CPM,AKB
1,PCA,PCB,TCA,TCB,EMA,EMB
000265      READ(5,101)PVA,PVB,PVC,PVD
000301      READ(5,101)SLA,SLB,SLC,SN1
000315      READ(5,101)DVA,DVB,DVC,DN1
000331      ALPHA=AK/(RHOL*CPL)
000333      PR=CPM*VIS/AKB
000336      WRITE(6,110)
000342      WRITE(6,113)(TITLE(I),I=1,18)
000354      WRITE(6,102)NRUN,MONTH,MDAY,MYEAR,KA,TEND,JA,P,TF,RS,T0,U,V,DT
000414      WRITE(6,103)ALPHA,RHOL,RR,PCA,PCB,TCA,TCB,EMA,EMB,EMV,AA,BB,VIS,PR
1,AKB,CPV,AK
000462      WRITE(6,104)PVA,PVB,PVC,PVD
000476      IF(SN1.LT..5)WRITE(6,114)SLA,SLB,SLC
000512      IF(SN1.GT..5)WRITE(6,115)SLA,SLB,SLC
000526      IF(DN1.GT..5)WRITE(6,116)
000534      IF(DN1.LT..5)WRITE(6,117)DVA,DVB,DVC
000550      TCRIT=100.*DT
000552      SSAMI=4.*3.14159*RS**3.*RHOL/3.
C
000560      DUM8=12.5663704*RHOL
000562      DUM2=.6*PR ** .333333
000566      DUM3=2.*ABS(U-V)
000572      DUM4=EMV/2./RR
000575      K=0
000576      J=0
000577      TIME=0.
000577      TS=T0
000601      TSSUM=TS
000602      TAVSUM=(TF+T0)*.5
000604      RSSUM=RS
000606      ROBAR=P*EMB/(RR*TAVSUM)
000611      VORSUM=VIS/ROBAR
000613      SCH=VORSUM/D(P,TAVSUM)
000617      DUM1=.6*SCH** .333333
000623      PVSUM=PV(T0)
000626      DUM=1.-PVSUM/P
000631      IF(DUM.LE.1.E-10)GO TO 11
000634      AKG=(2.+DUM1*SQRT(RS*DUM3/VORSUM))*D(P,T0)*DUM4/RS/TAVSUM
000652      RSDOT= AKG*P*ALOG(1.-PVSUM/P)/RHOL
000663      WDOT=DUM8*RS**2*RSDOT
000667      WRITE(6,107)
000672      WRITE(6,106)K,TIME,RS,RSDOT,WDOT,TS
000712      RSINT=0.
000713      VORINT=0.
000713      TAVINT=0.
000714      PVINT=0.
000715      TSINT=0.
C
000717      2 K=K+1
000721      J=J+1
000722      TIME=TIME+DT
000724      KIP=1
000724      ME=1
000725      TS1(1)=TS
000727      TS1(2)=0.9*TS
000731      3 TAV=(TF+TS1(ME))* .5

```

```

000734 DD=D(P,TAV)
000740 ROAV=P*EMB/(RR*TAV)
000744 VOR=VIS/ROAV
000745 RS1=RS+RSDOT*DT
000751 IF(RS1.LE.0.0) GO TO 12
000753 RSBAR=(RSINT+RS1*DT)/TIME
000755 VORBAR=(VORINT+VOR*DT)/TIME
000760 IF(ABS(PVA-PVB/(TS1(ME)-PVC)+PVD*TS1(ME)).GT.300.) GO TO 13
000773 PVBAR=(PVINT+PV(TS1(ME))*DT)/TIME
001001 TAVBAR=(TAVINT+TS1(ME)*DT)/TIME
001004 TSBAR=(TSINT+TS1(ME)*DT)/TIME
001010 H=AKB/2./RSBAR*(2.+DUM2*SQRT(DUM3*RSBAR/VORBAR))
001021 DUM1=.6*(VORBAR/D(P,TAVBAR))**.333333
001031 AKGBAR=(2.+DUM1*SQRT(RSBAR*DUM3/VORBAR))*D(P,TAVBAR)*DUM4/RSBAR/TA
1VBAR
001050 DUM=1.-PVBAR/P
001054 IF(DUM.LE.1.E-10)GO TO 11
001057 DUM=(AKGBAR*P*ALOG(DUM))/AK
001063 C=H/AK+CPV*DUM
001067 IF(SN1.LT..5)HEVAP=SL2(TSBAR)
001074 IF(SN1.GT..5)HEVAP=SL1(TSBAR)
001101 B=-HEVAP*DUM-C*TF
001105 A=C-1./RSBAR
001107 DUM5=SQRT(ALPHA*TIME)/RSBAR
001114 DUM6=DUM5*A*RSBAR
001116 DUM7=DUM6**2
001117 DUM9=EXP(DUM7)
001121 TSA=T0+(T0+(B+T0/RSBAR)/A)*(ERFC(1./DUM5)-ERFC(0.))+DUM9*(ERFC(DUM6
1)-EXP(2.*A*RSBAR)*ERFC(1./DUM5+DUM6))
001161 ERR(ME)=TSA-TS1(ME)
001164 IF(ABS(ERR(ME)).LE.1.E-4)GO TO 4
001170 IF(ME.EQ.2)GO TO 5
001172 ME=2
001173 GO TO 3
001173 5 TSAA=TS1(1)-ERR(1)*(TS1(2)-TS1(1))/(ERR(2)-ERR(1))
001202 ERR(1)=ERR(2)
001203 TS1(1)=TS1(2)
001204 TS1(2)=TSAA
001206 KIP=KIP+1
001207 IF(KIP.LE.20)GO TO 3
001212 WRITE(6,105)
001215 GO TO 1

C
001216 4 RS=RS1
001217 RSINT=RSINT+RS*DT
001222 TS=TS1(ME)
001224 DUM1=.6*(VOR/DD)**.333333
001231 AKG=(2.+DUM1*SQRT(RS*DUM3/VOR))*DD*DUM4/RS/TAV
001244 IF(ABS(PVA-PVB/(TS-PVC)+PVD*TS).GT.300.) GO TO 13
001256 DUM=1.-PV(TS)/P
001262 IF(DUM.LE.1.E-10)GO TO 11
001265 RSDOT=AKG*P*ALOG(DUM)/RHOL
001271 TAVINT=TAVINT+TAV*DT
001275 VORINT=VORINT+VOR*DT
001277 TSINT=TSINT+TS*DT
001302 PVINT=PVINT+PV(TS)*DT
001306 WDOT=DUM8*RS**2*RSDOT
001310 SCH=VOR/DD

```

```

001312      REY=2.*RS*ABS(U-V)/VOR
001320      SSAM=4.*3.14159*RS**3.*RHOL/3.
001325      RAT=SSAM/SSAMI
001327      IF (TIME.GT.TCRIT.AND.NP.GT.0) GO TO 10
001341      WRITE (6,106) K, TIME, RS, RSDOT, WDOT, TS, AKG, H, SCH, REY, DD, RAT
001374      10 IF (RS.LE.0.) TEND=TIME
001400      IF (J.LT.JA.AND.TIME.LT.TEND) GO TO 6

      C
001412      8 J=0
001413      TT=TIME/DT+.5
001416      IF (TT.GT.100.) DT=10.*DT
001422      WRITE (6,109) K
001430      DO 7 N=1,10
001432      R=.1*FLOAT(N)
001435      DOM1=(1.+R)/2./DUM5
001440      DOM2=(1.-R)/2./DUM5
001442      T=T0+(T0+(B+T0/RSBAR)/A)/R*(ERFC(DOM1)-ERFC(DOM2)+DUM9*(EXP(A*RSBA
1R*(1.-R))*ERFC(DOM2+DUM6)-EXP(A*RSBAR*(1.+R))*ERFC(DOM1+DUM6)))
001512      7 WRITE (6,108) R, T
001524      WRITE (6,107)
001530      IF (NP.LT.0) GO TO 6
001532      WRITE (6,106) K, TIME, RS, RSDOT, WDOT, TS, AKG, H, SCH, REY, DD, RAT
001565      IF (RAT.LT..9) GO TO 14
001570      6 IF (K.GT.KA.OR.TIME.GE.TEND) GO TO 1
001603      GO TO 2
001603      11 WRITE (6,111)
001607      GO TO 1
001610      12 WRITE (6,118) TIME
001616      GO TO 1
001617      13 WRITE (6,119)
001623      GO TO 1
001624      14 WRITE (6,120)
001630      GO TO 1

      C
001631      END

```



## APPENDIX D

### SYMBOLS

|                 |   |
|-----------------|---|
| A               | factor, Eq. (12)  |
| $A_d$           | droplet surface area  |
| B               | film thickness factor, Eq. (12), Eq. (A-7a)                   |
| a,b,c           | constants, Eq. (6)  |
| C               | constant, Eq. (13), (A-9a)                                    |
| $(\bar{c}_p)_v$ | average droplet vapor specific heat                           |
| D               | molecular diffusion coefficient, factor, Eq. (A-7b), diameter |
| h               | film heat transfer coefficient, enthalpy                      |
| K               | evaporative constant, Eq. (20)                                |
| $K_g$           | mass transfer coefficient                                     |
| $K_m$           | thermal conductivity of vapor-gas mixture                     |
| M               | molecular weight  |
| Pr              | Prandtl number  |
| p               | pressure; transform variable                                  |
| q               | defined in Eq. (A-5c), heat transfer                          |
| R               | ratio $r/r_s$   |
| Re              | Reynolds number   |
| $R_o$           | universal gas constant  |
| r               | droplet radius  |
| $\dot{r}_s$     | regression rate of droplet surface                            |
| s               | entropy   |
| Sc              | Schmidt number  |
| T               | temperature   |
| $\bar{T}$       | average temperature $[T_f + T_s]/2$                           |
| t               | time  |
| U               | function defined in Eq. (A-1)                                 |
| u               | gas velocity  |
| v               | droplet velocity  |
| $\dot{w}$       | mass evaporation rate   |
| x               | mole fraction   |
| y               | mass fraction   |
| $\alpha$        | thermal diffusivity   |
| $\gamma$        | exponent, Eq. (7)   |
| $\rho$          | gas density   |
| $\kappa$        | droplet thermal conductivity                                  |
| $\lambda$       | heat of vaporization  |

|             |                    |
|-------------|--------------------|
| $\lambda_0$ | constant, Eq. (7)  |
| $\mu$       | viscosity          |
| $\rho$      | density            |
| $\tau$      | $\alpha t / r_s^2$ |
| $\theta$    | transport property |
| $\varphi$   | Eq. (B-5)          |

### Subscripts

|           |  |
|-----------|--|
| m         | mixture  |
| sh        | superheat                                      |
| $\lambda$ | evaporation                                    |
| $l$       | liquid   |
| T         | total  |
| s         | surface  |
| o         | initial, reference                             |
| f         | flame  |
| v         | vapor  |
| c         | critical, chamber                              |
| r         | radius, differentiation with respect to radius |
| t         | time, differentiation with respect to time     |
| i,j       | specie index                                   |

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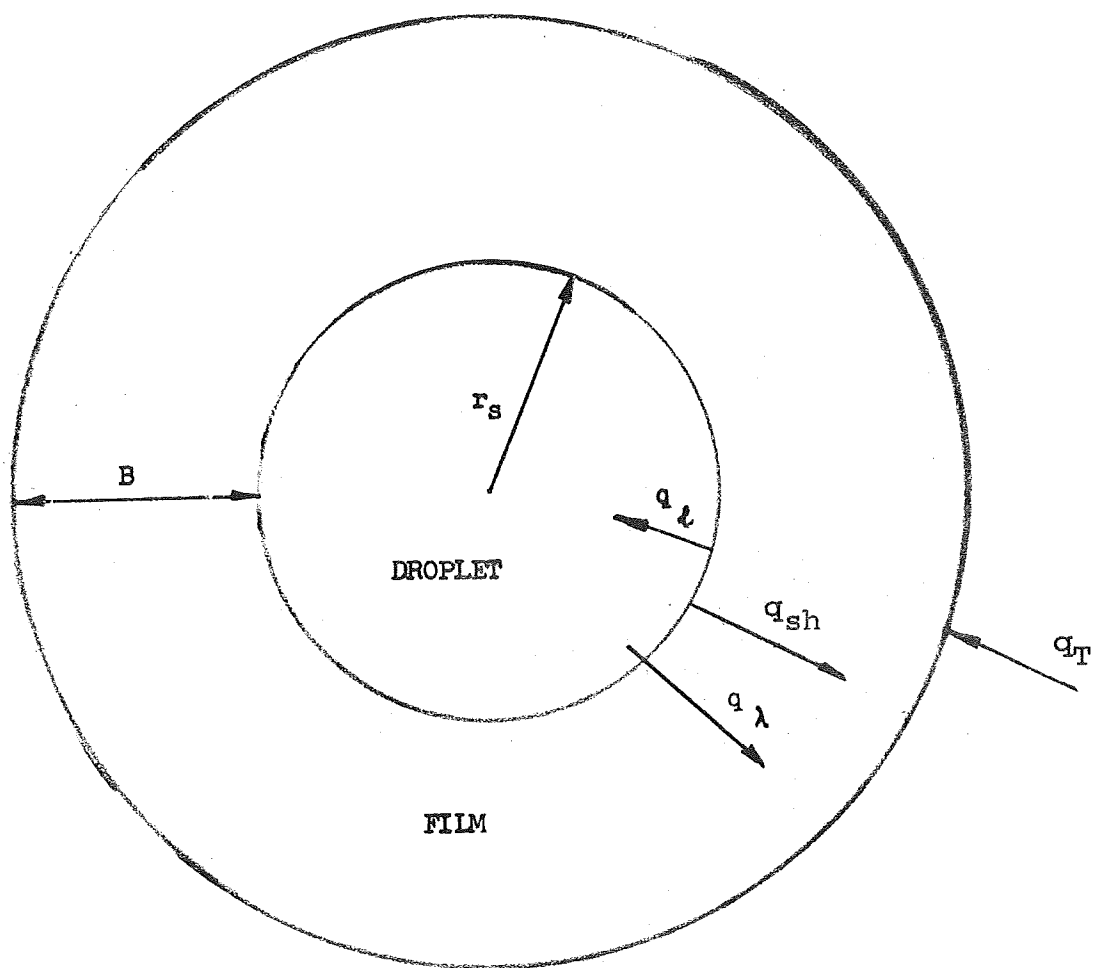


FIG. 1. SCHEMATIC DIAGRAM OF HEAT TRANSFER TO VAPOR FILM AND LIQUID DROPLET

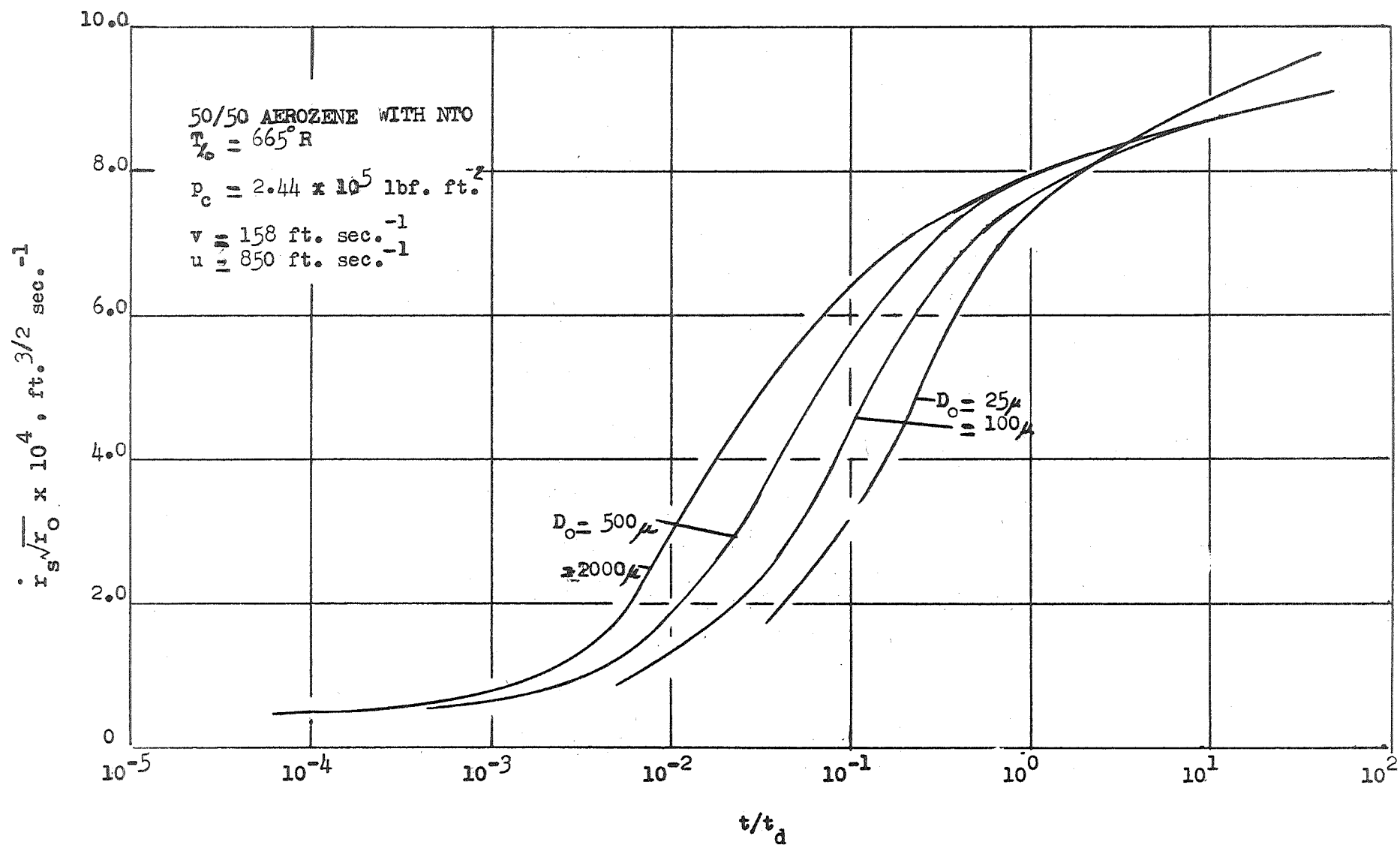


FIG. 2a. SURFACE REGRESSION RATE VS. TIME FOR VARIATION IN INITIAL DROPLET RADIUS, 50/50 AEROZENE WITH NTO

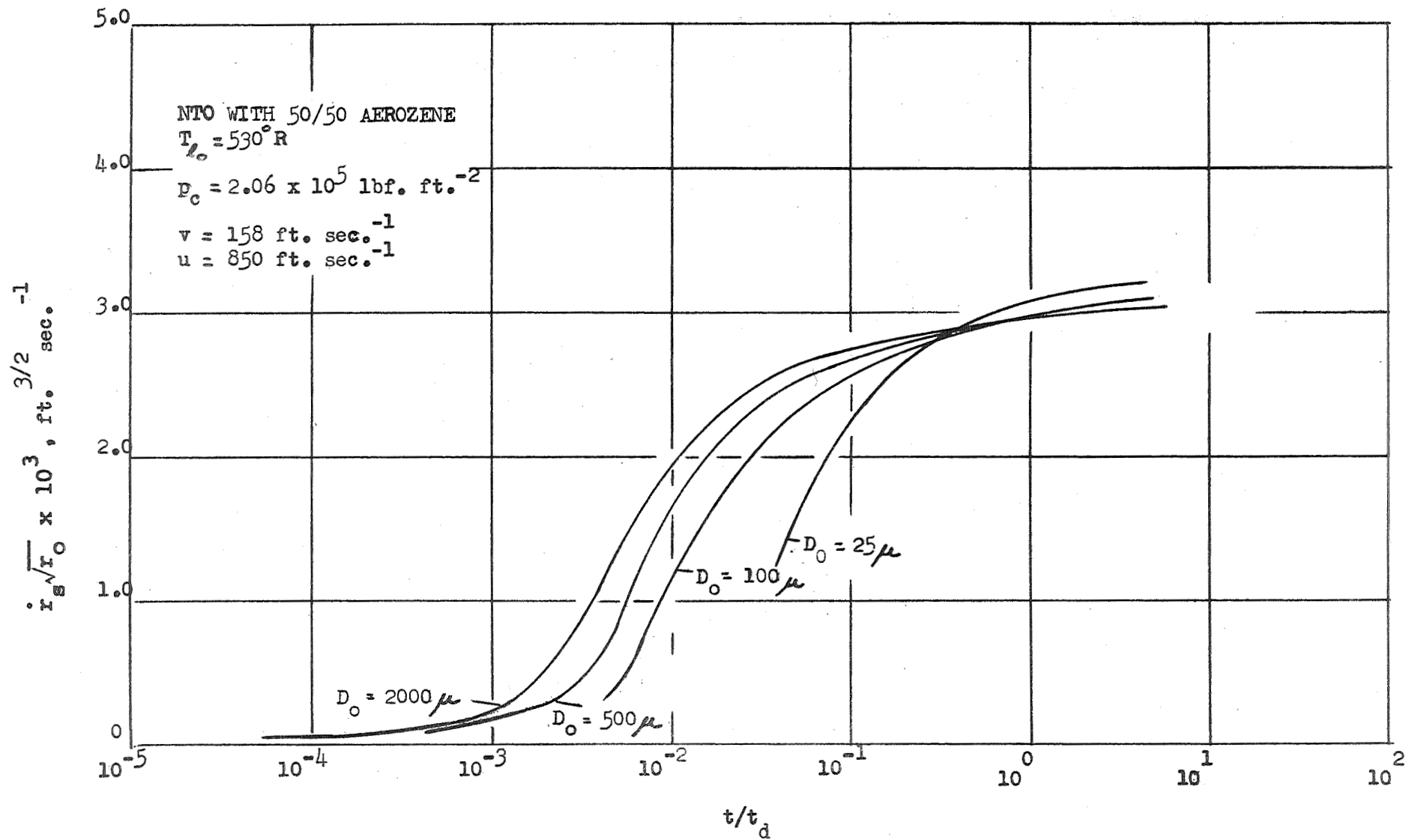


FIG. 2b. SURFACE REGRESSION RATE VS. TIME FOR VARIATION IN INITIAL DROPLET RADIUS, NTO WITH 50/50 AEROZINE

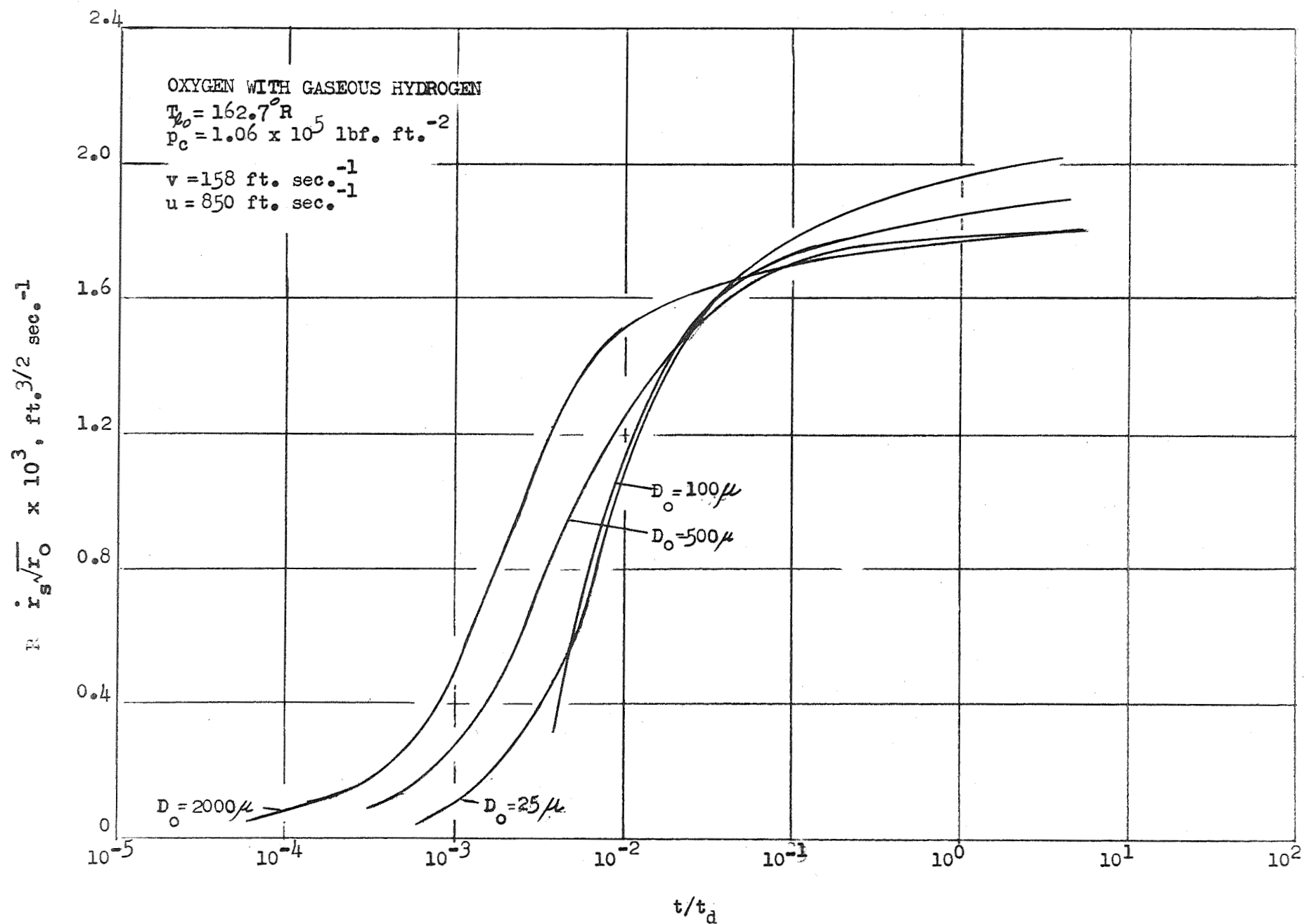


FIG. 2c. SURFACE REGRESSION RATE VS. TIME FOR VARIATION IN INITIAL DROPLET RADIUS, OXYGEN WITH GASEOUS HYDROGEN

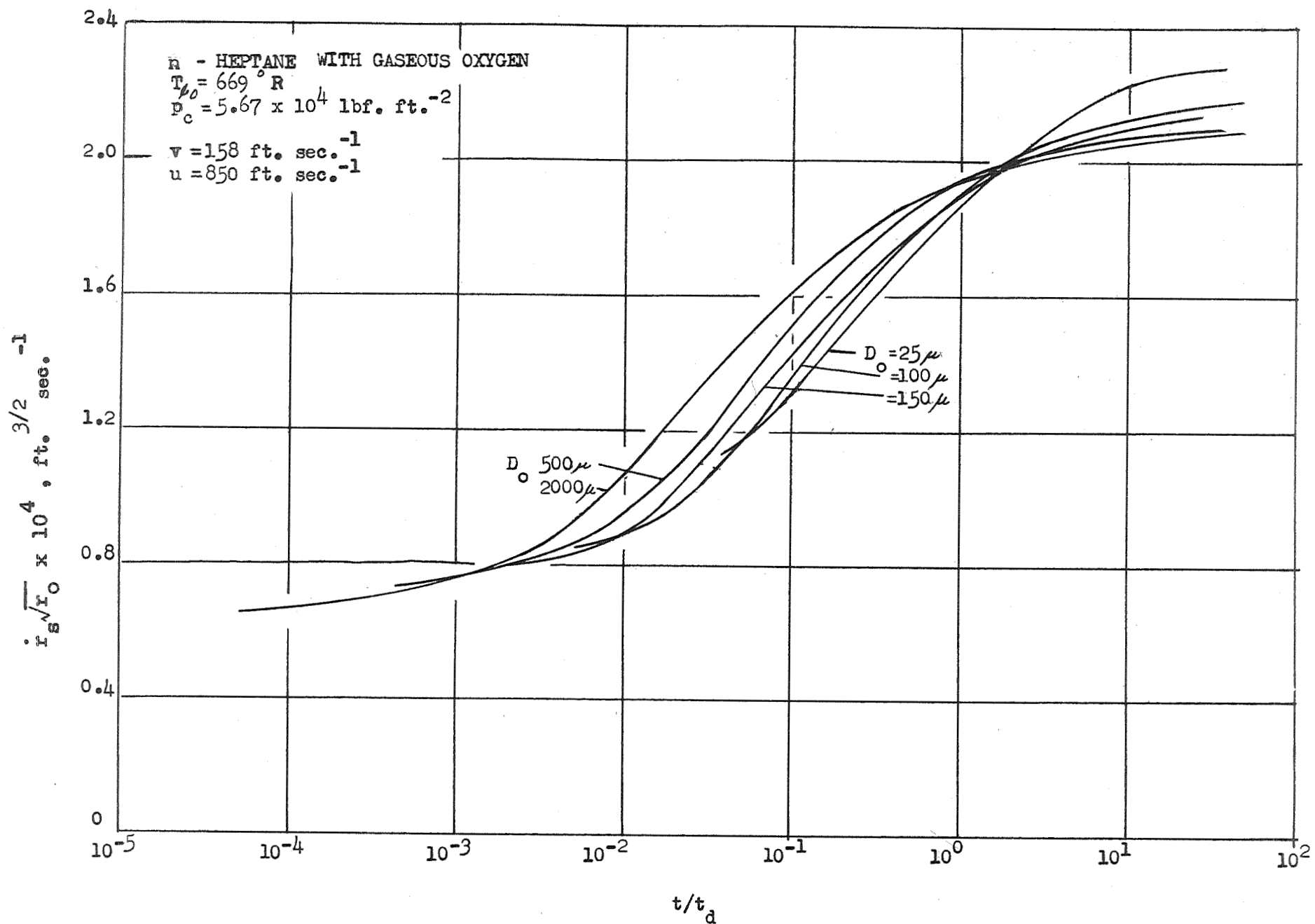


FIG. 2d. SURFACE REGRESSION RATE VS. TIME FOR VARIATION IN INITIAL DROPLET RADIUS, n-HEPTANE WITH GASEOUS OXYGEN



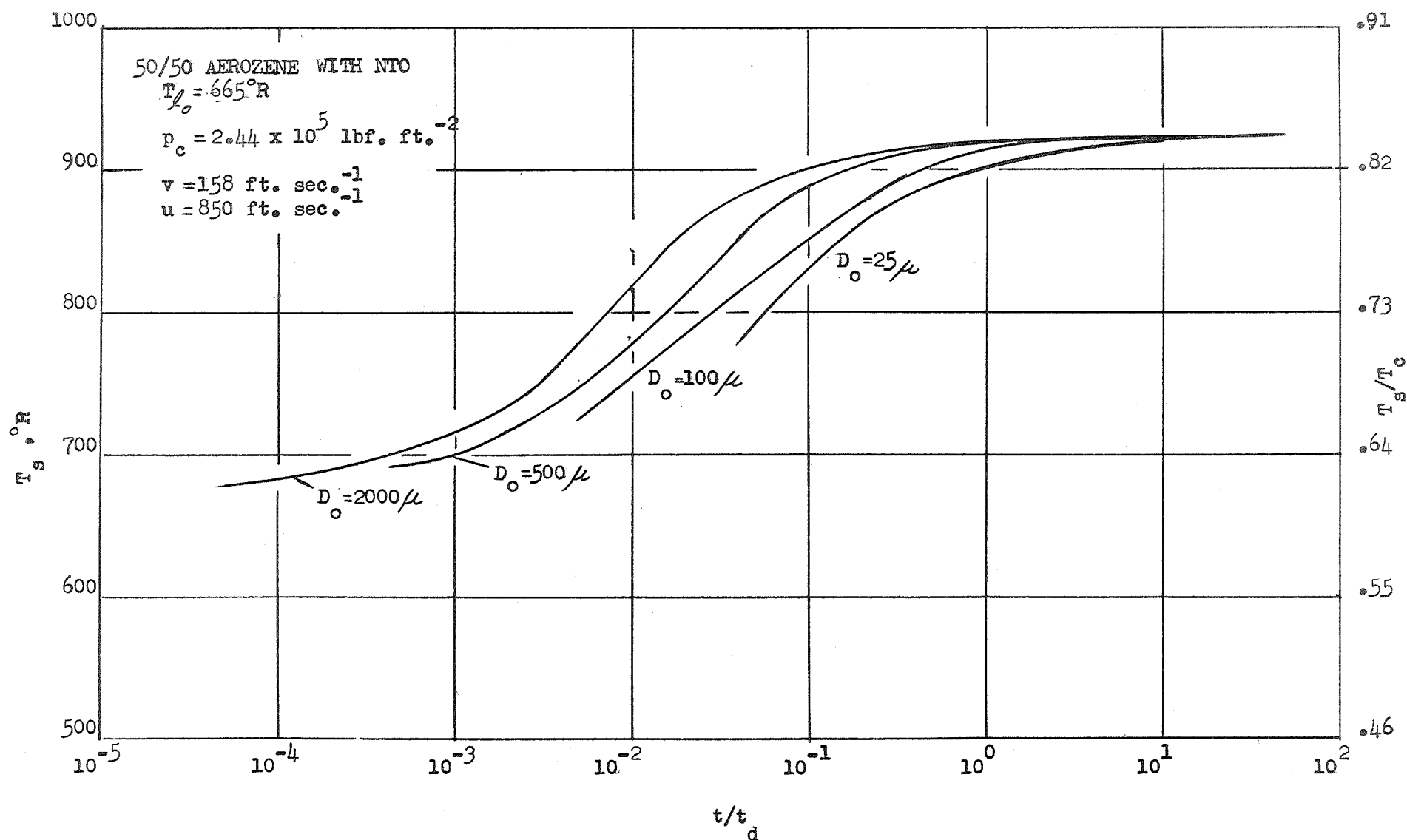


FIG. 3a. SURFACE TEMPERATURE VS. TIME FOR VARIATION IN INITIAL DROPLET RADIUS, 50/50 AEROZINE WITH NTO

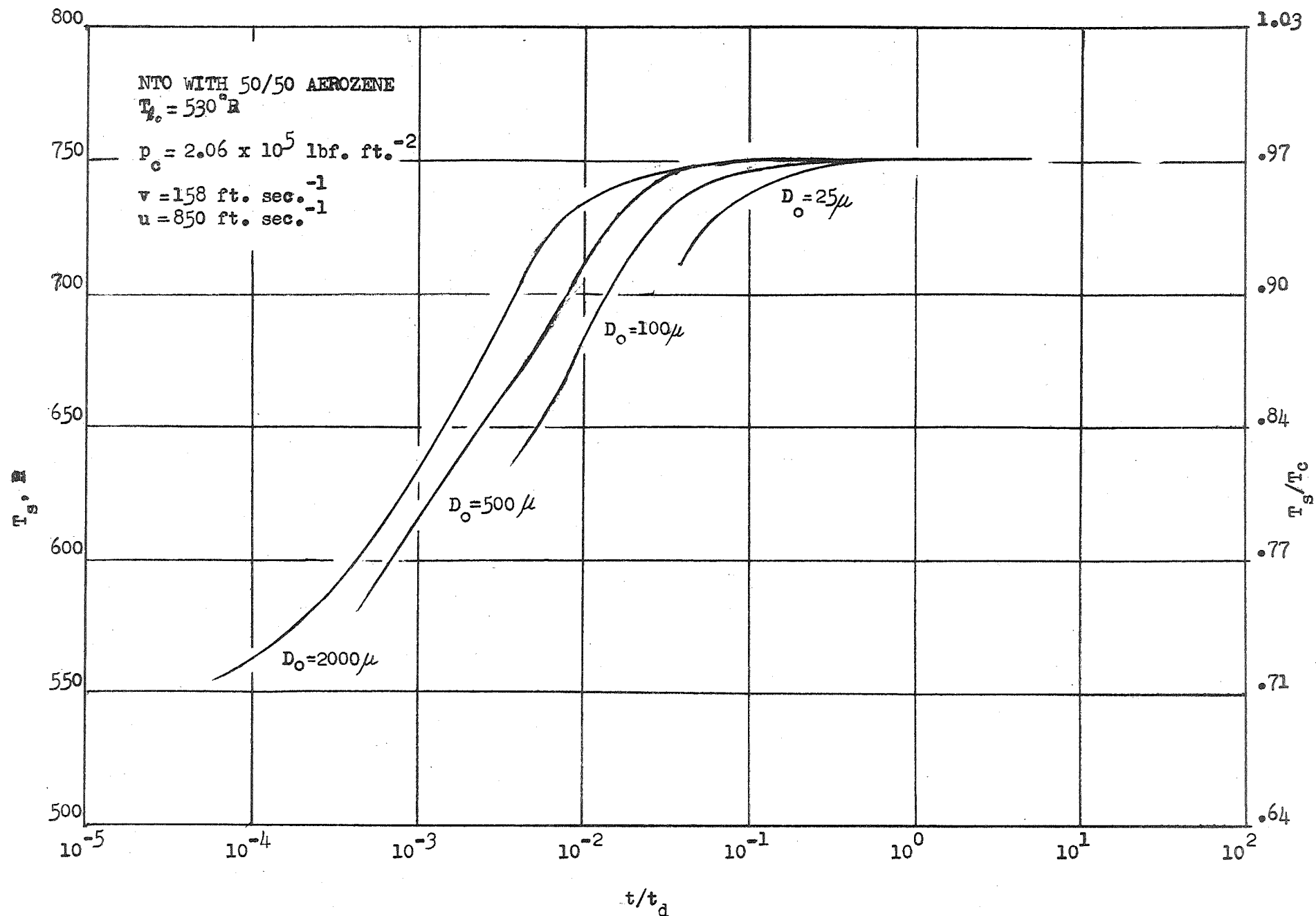


FIG. 3b. SURFACE TEMPERATURE VS. TIME FOR VARIATION IN INITIAL DROPLET RADIUS, NTO WITH 50/50 AEROZINE

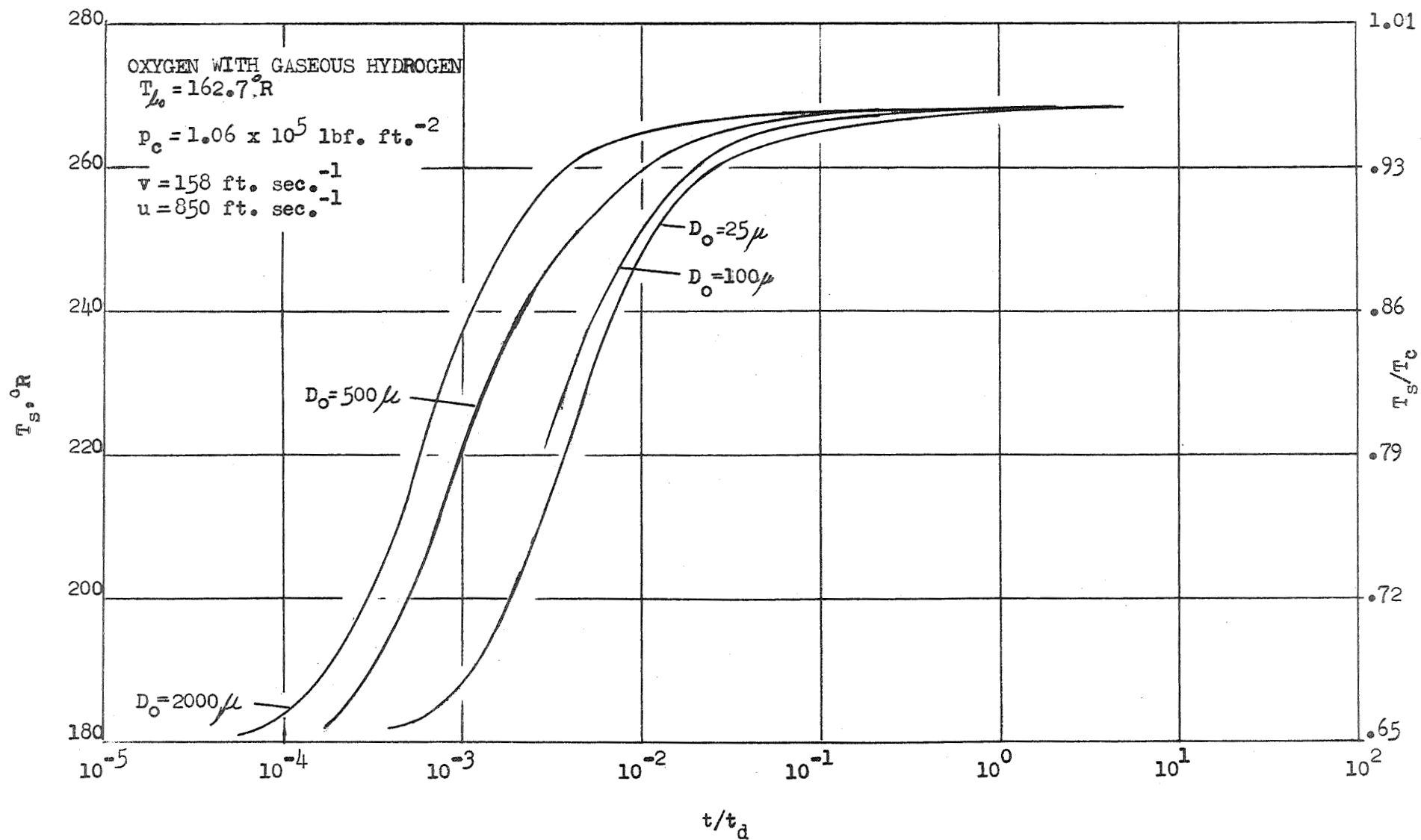


FIG. 3c. SURFACE TEMPERATURE VS. TIME FOR VARIATION IN INITIAL DROPLET RADIUS, OXYGEN WITH GASEOUS HYDROGEN

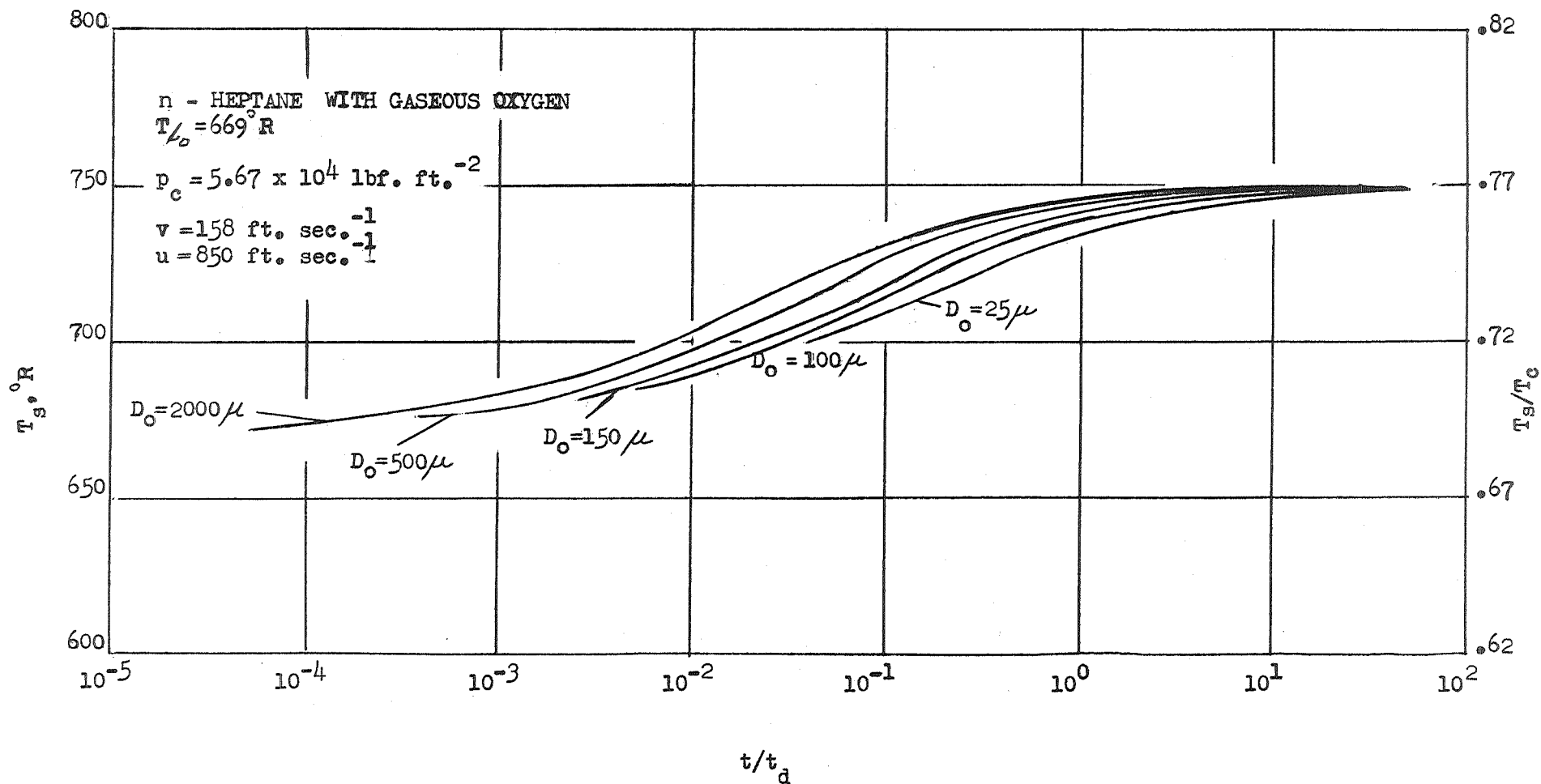


FIG. 3d. SURFACE TEMPERATURE VS. TIME FOR VARIATION IN INITIAL DROPLET RADIUS, n-HEPTANE WITH GASEOUS OXYGEN

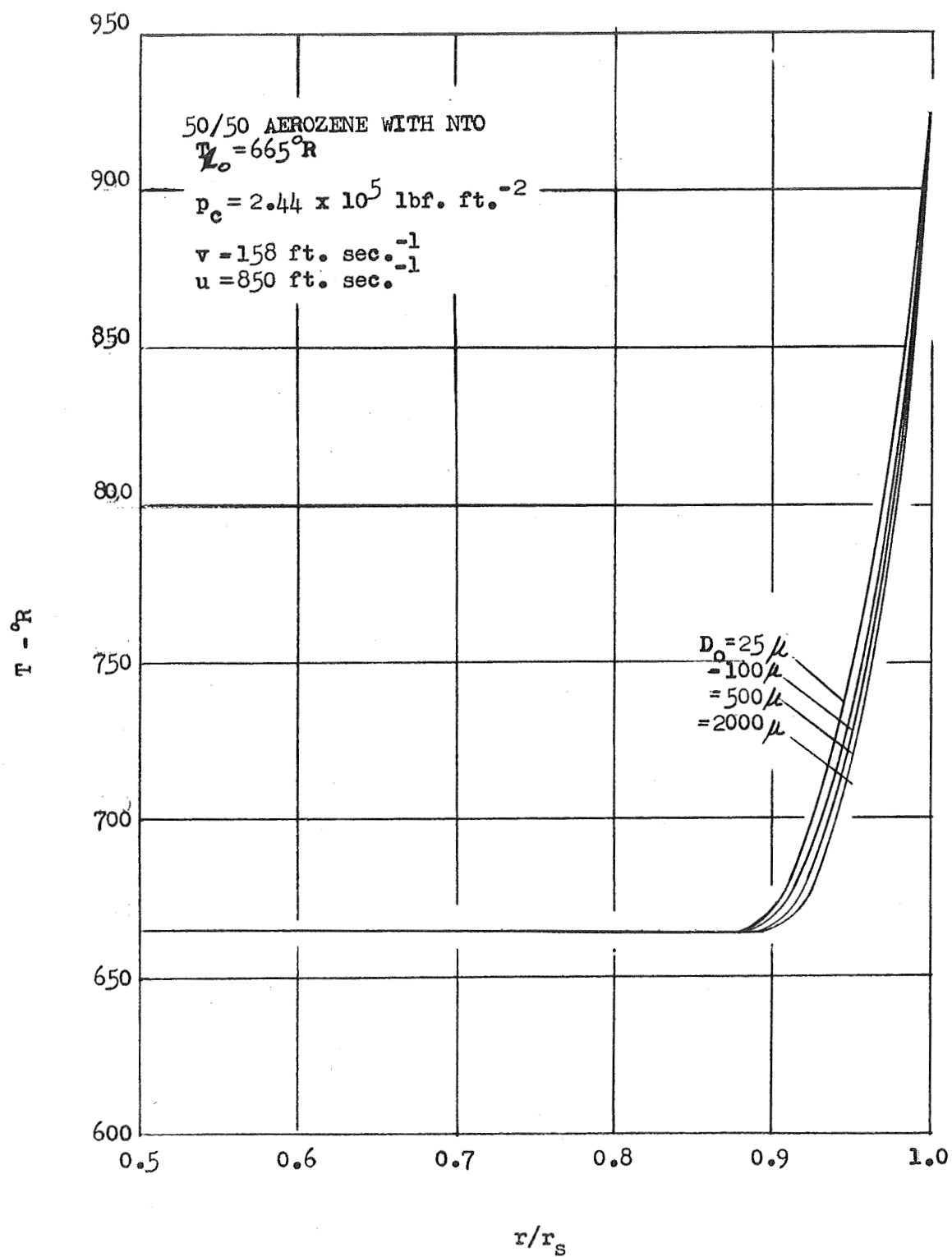


FIG. 4a. DROPLET TEMPERATURE DISTRIBUTION FOR VARIATION IN INITIAL DROPLET RADIUS, AT A TIME  $10^{-4}$  SECOND, 50/50 AEROSOL WITH NTO

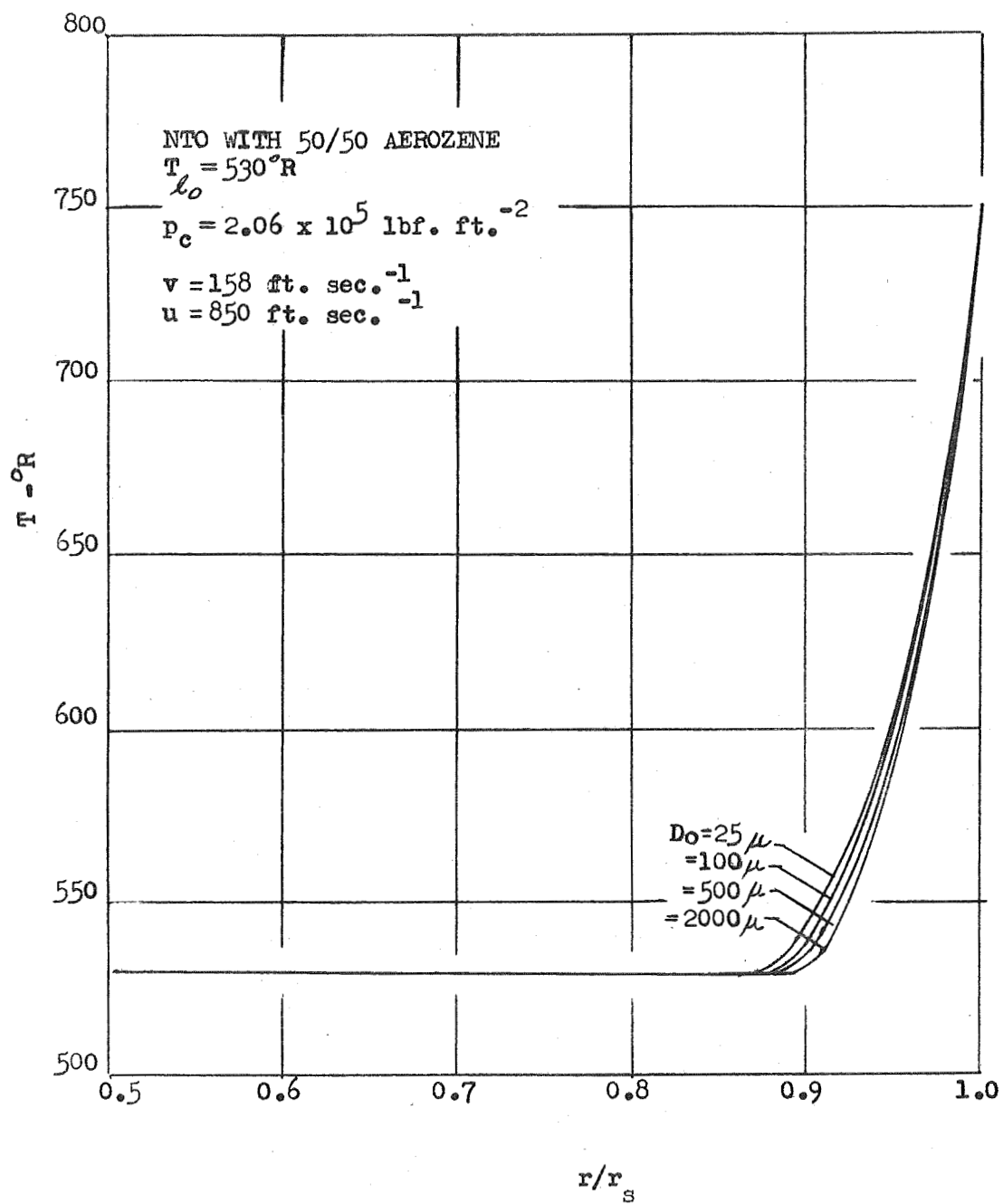


FIG. 4b. DROPLET TEMPERATURE DISTRIBUTION FOR VARIATION IN INITIAL DROPLET RADIUS, AT A TIME OF  $10^{-4}$  SECOND, NTO WITH 50/50 AEROZINE

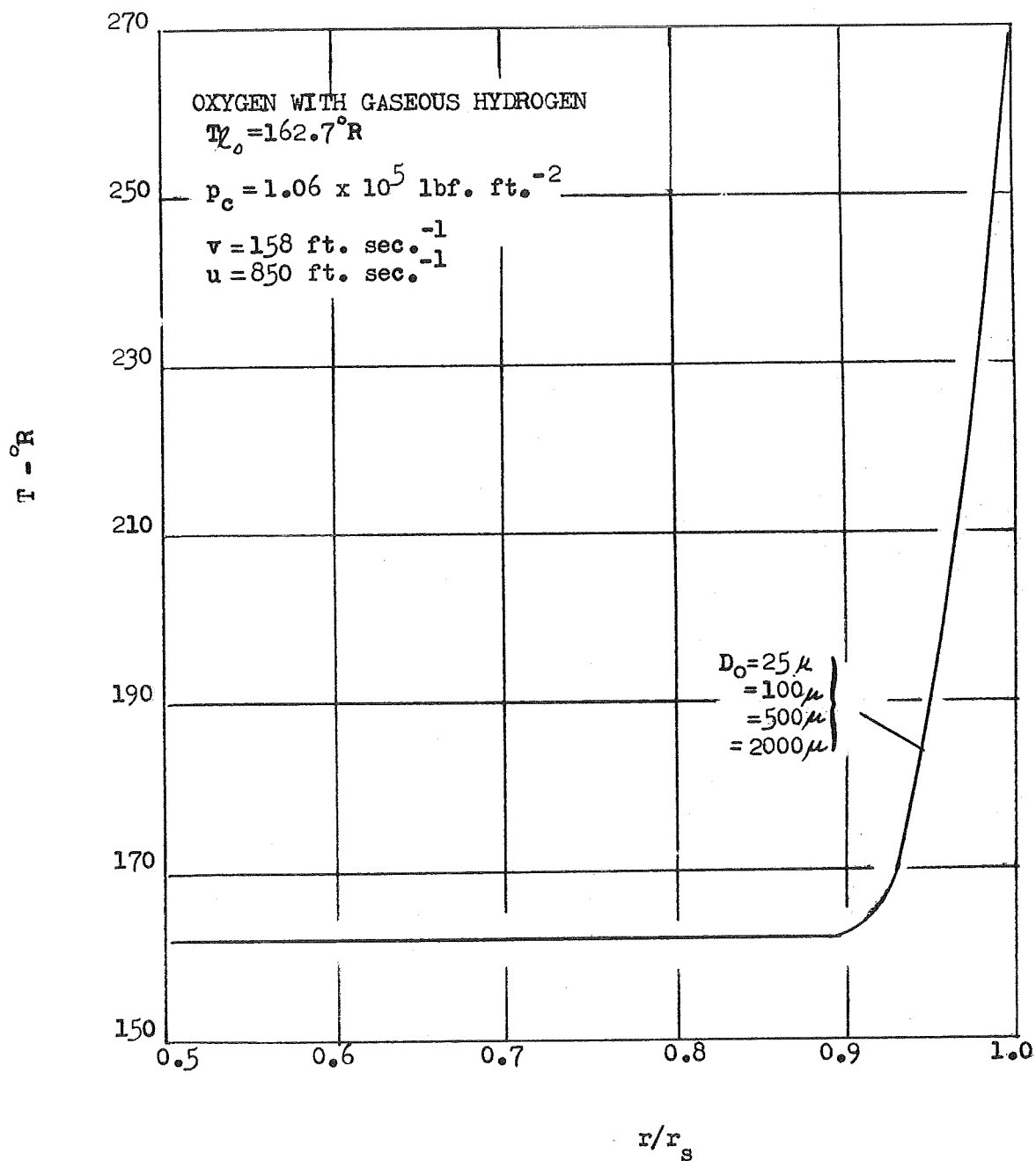


FIG. 4c. DROPLET TEMPERATURE DISTRIBUTION FOR VARIATION IN INITIAL DROPLET RADIUS, AT A TIME OF  $10^{-4}$  SECOND, OXYGEN WITH GASEOUS HYDROGEN

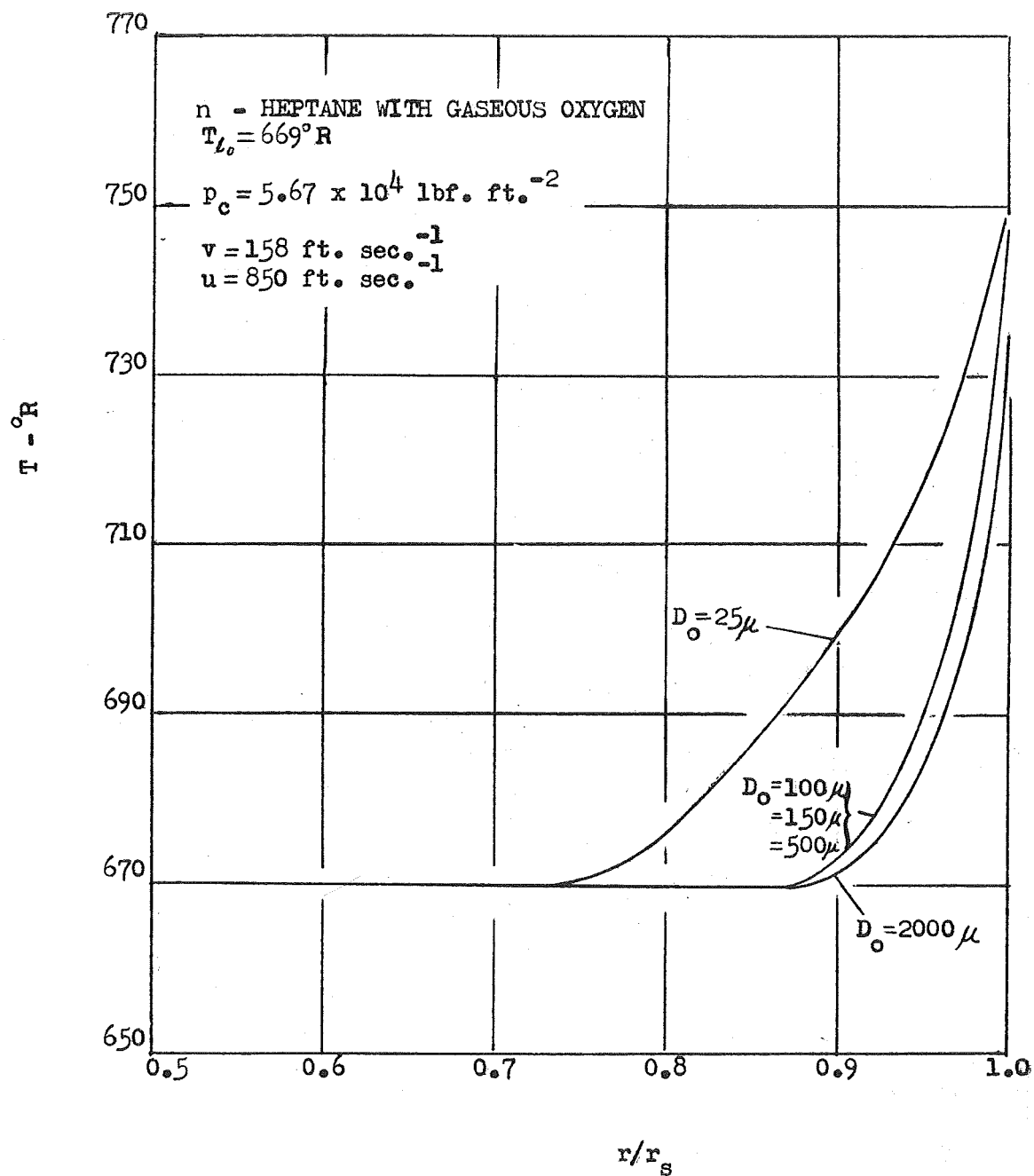


FIG. 4d/ DROPLET TEMPERATURE DISTRIBUTION FOR VARIATION  
 IN INITIAL DROPLET RADIUS, AT A TIME OF  $10^{-4}$  SECOND,  
 n-HEPTANE WITH GASEOUS OXYGEN



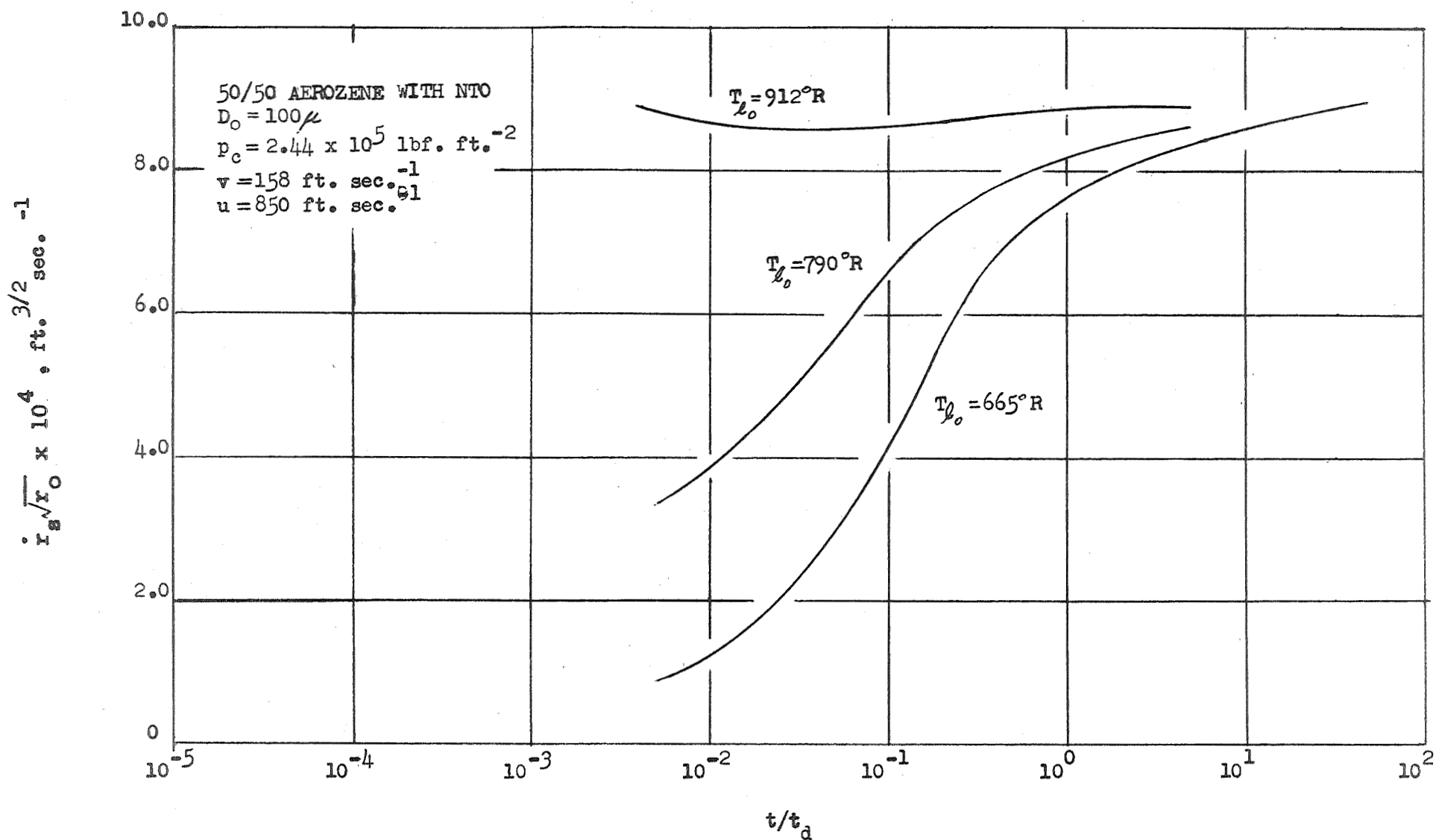


FIG. 5a . SURFACE REGRESSION RATE VS. TIME FOR VARIATION IN INITIAL DROPLET TEMPERATURE, 50/50 AEROZENE WITH NTO

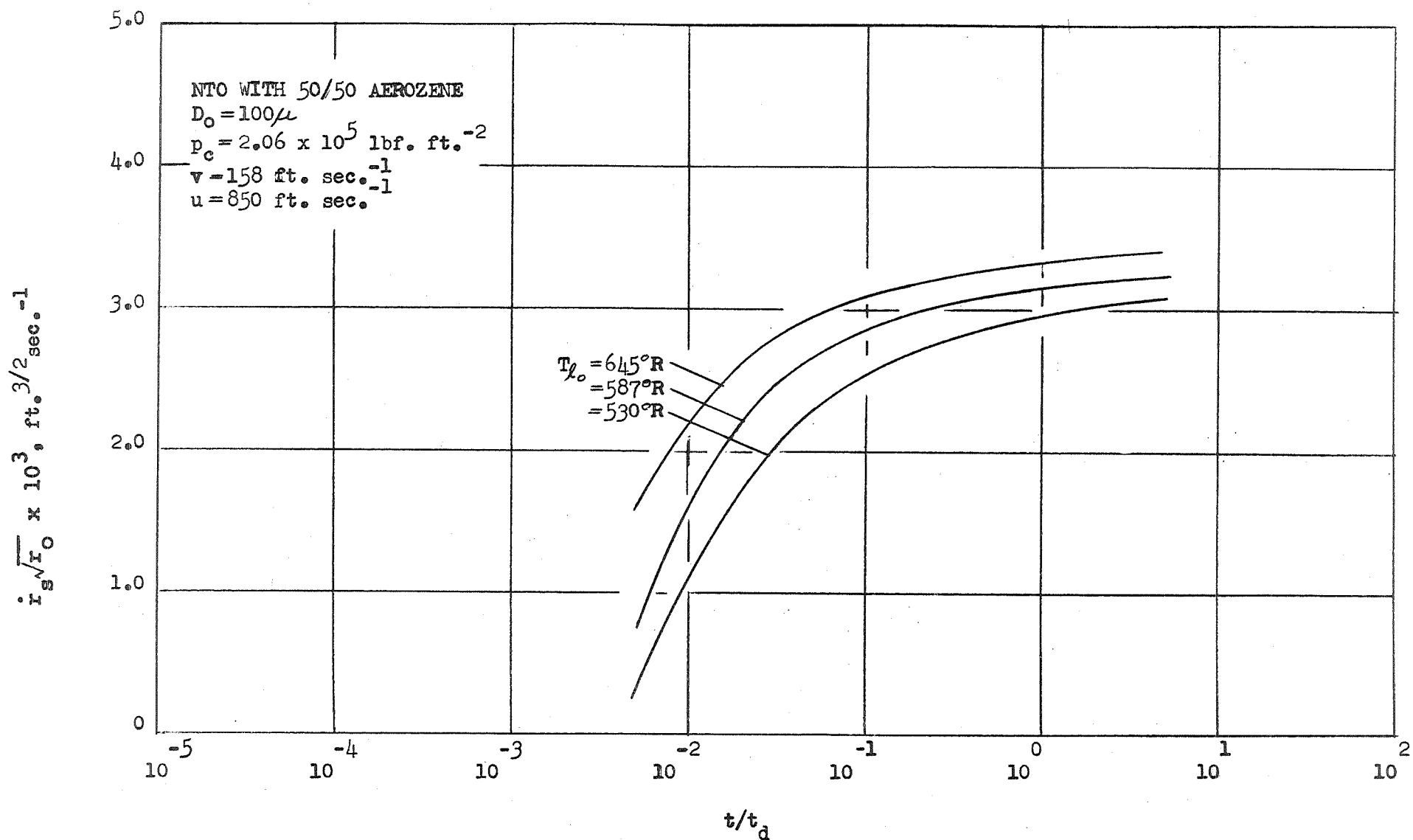


FIG. 5b. SURFACE REGRESSION RATE VS. TIME FOR VARIATION IN INITIAL DROPLET TEMPERATURE, NTO WITH 50/50 AEROZINE

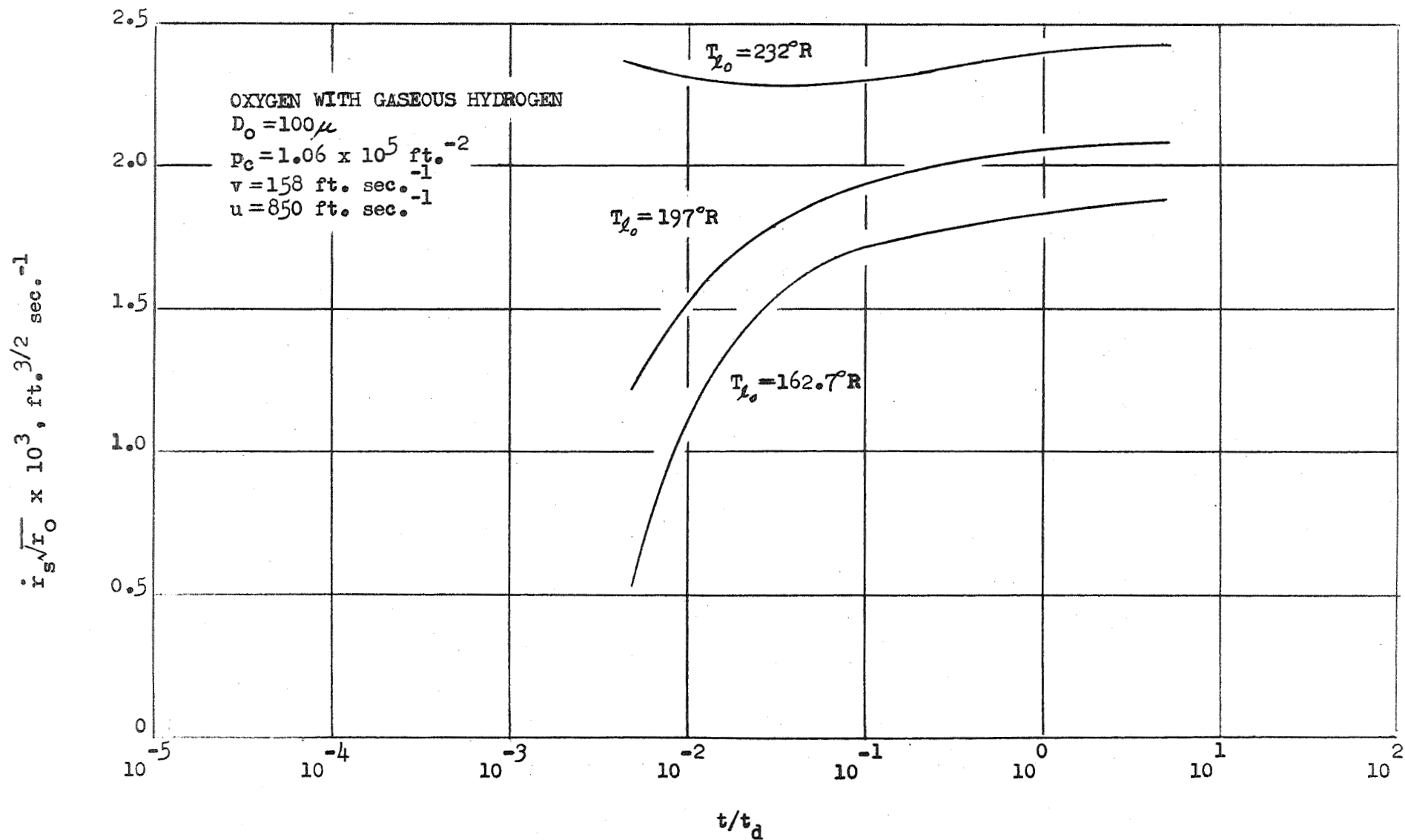


FIG. 5c . SURFACE REGRESSION RATE VS. TIME FOR VARIATION IN INITIAL DROPLET TEMPERATURE, OXYGEN WITH GASEOUS HYDROGEN

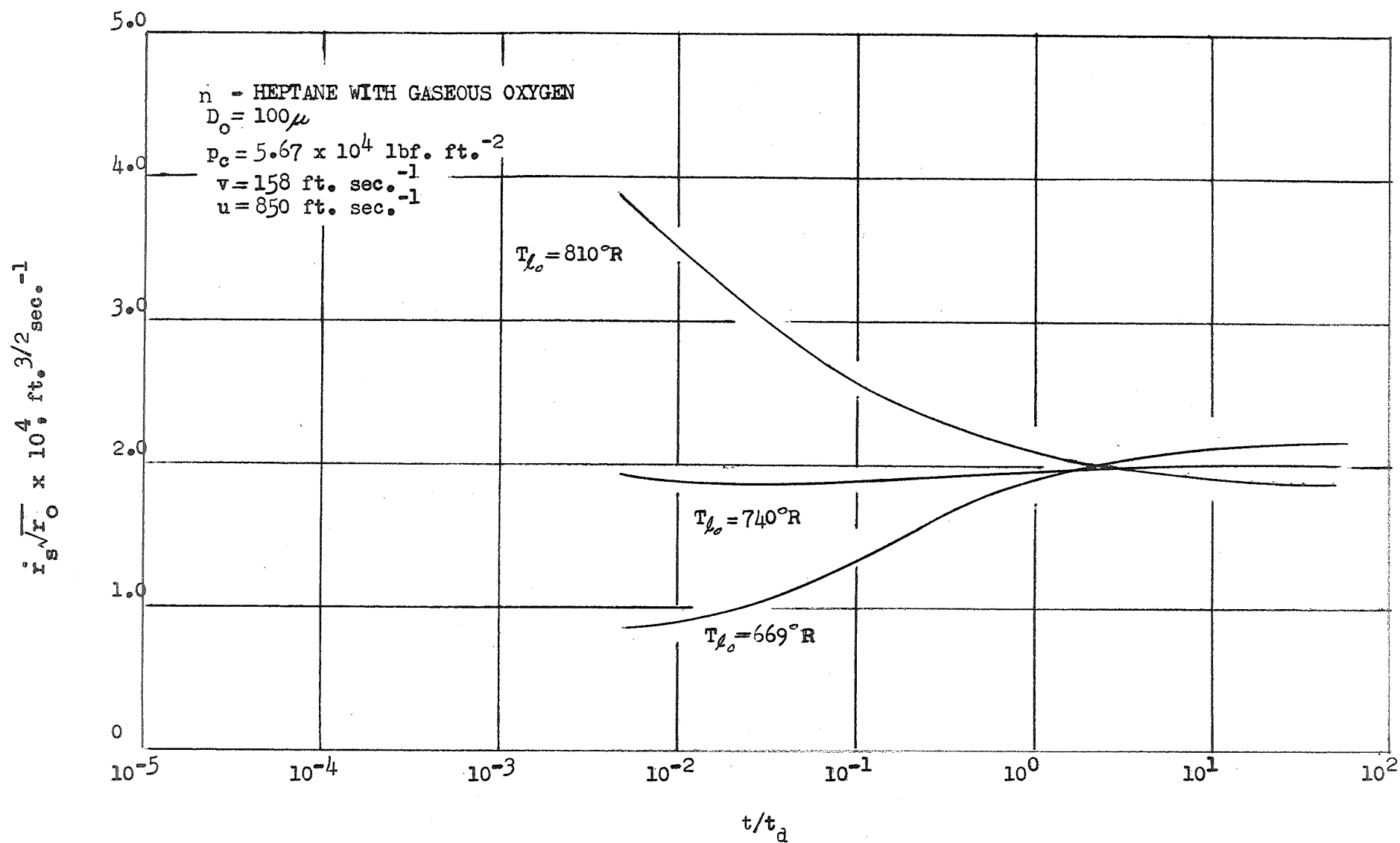


FIG. 5 d. SURFACE REGRESSION RATE VS. TIME FOR VARIATION IN INITIAL DROPLET TEMPERATURE, n-HEPTANE WITH GASEOUS OXYGEN

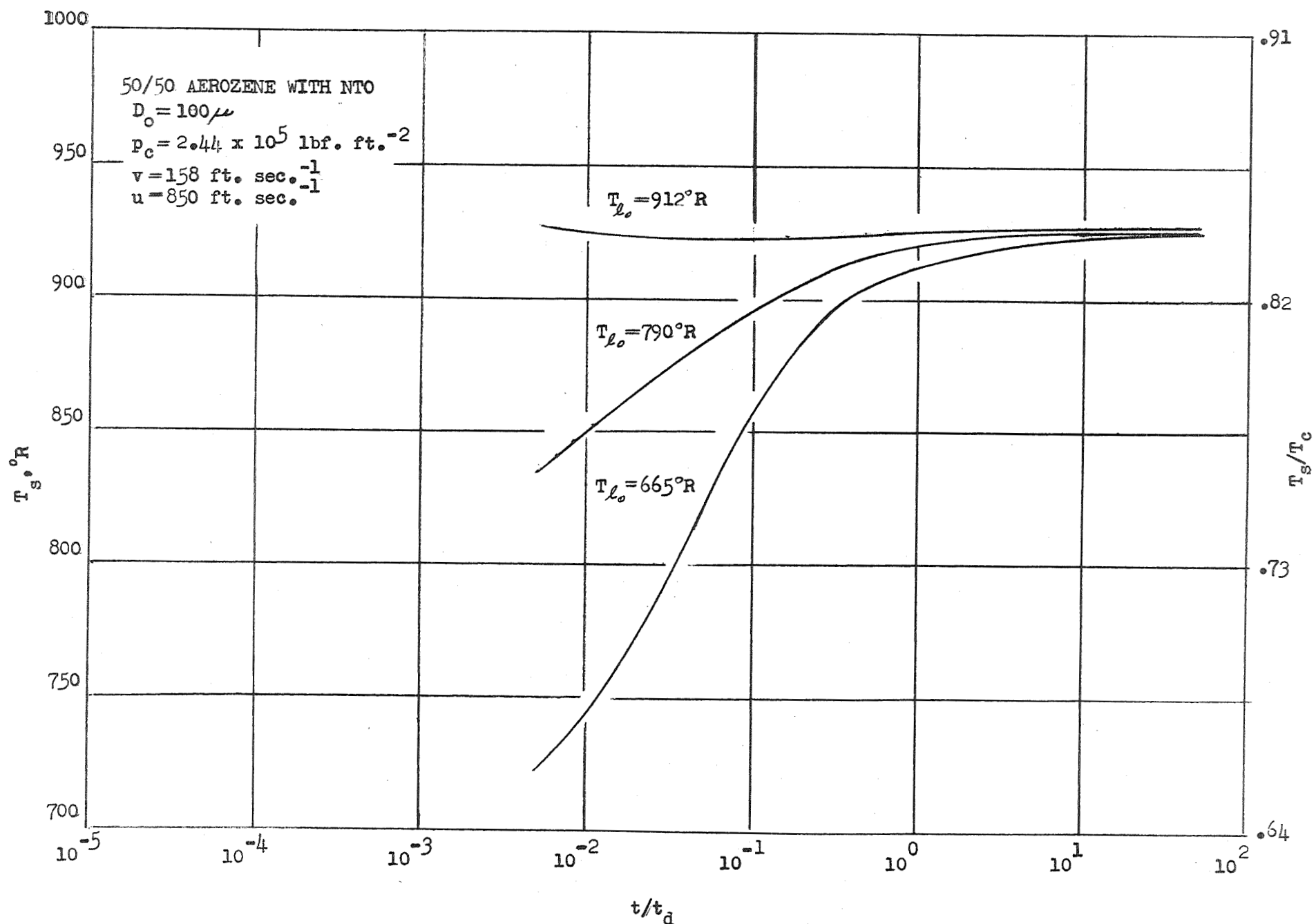


FIG. 6a. SURFACE TEMPERATURE VS. TIME FOR VARIATION IN INITIAL DROPLET TEMPERATURE, 50/50 AEROZINE WITH NTO

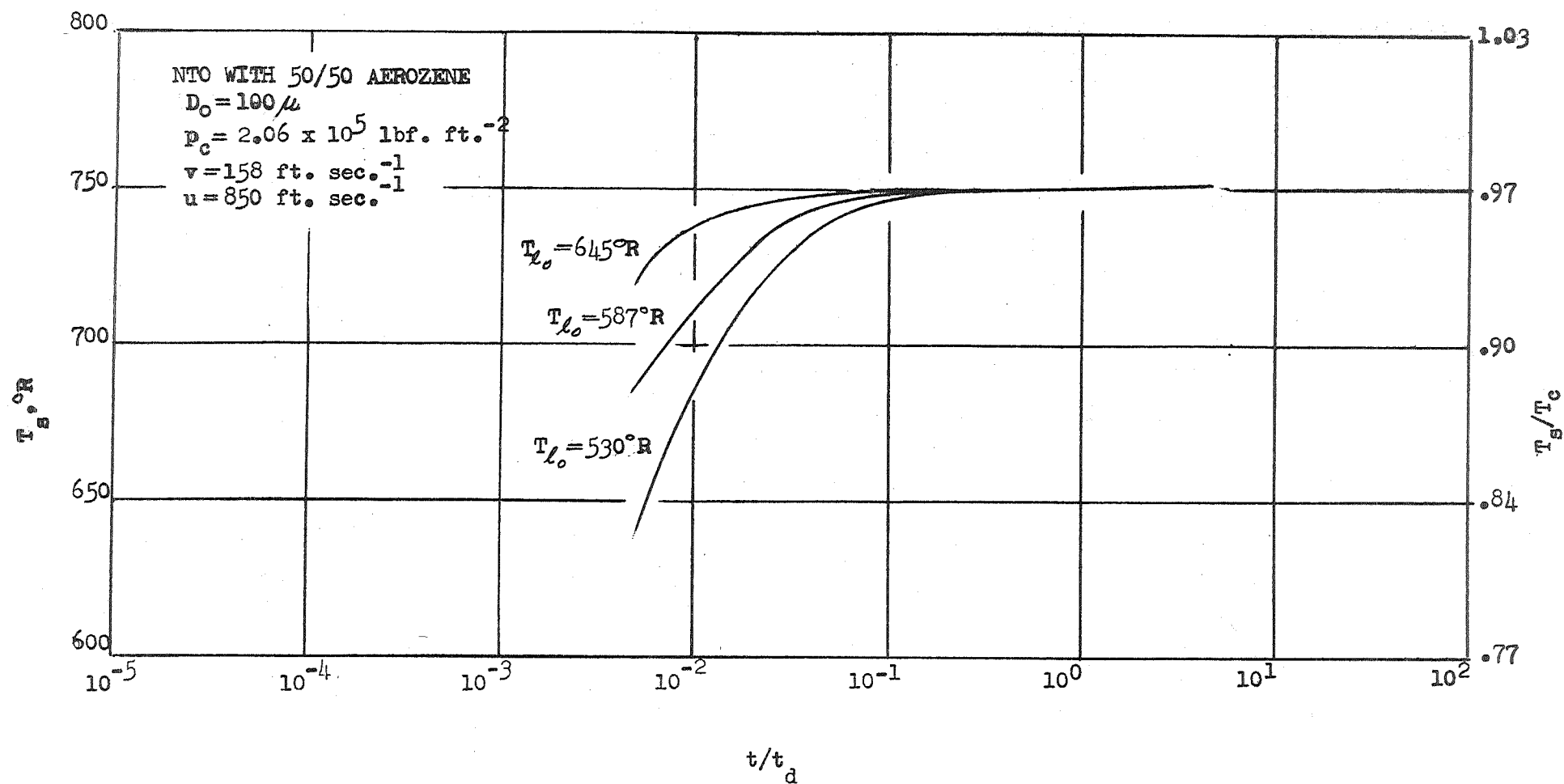


FIG. 6b. SURFACE TEMPERATURE VS. TIME FOR VARIATION IN INITIAL DROPLET TEMPERATURE, NTO WITH 50/50 AEROZINE

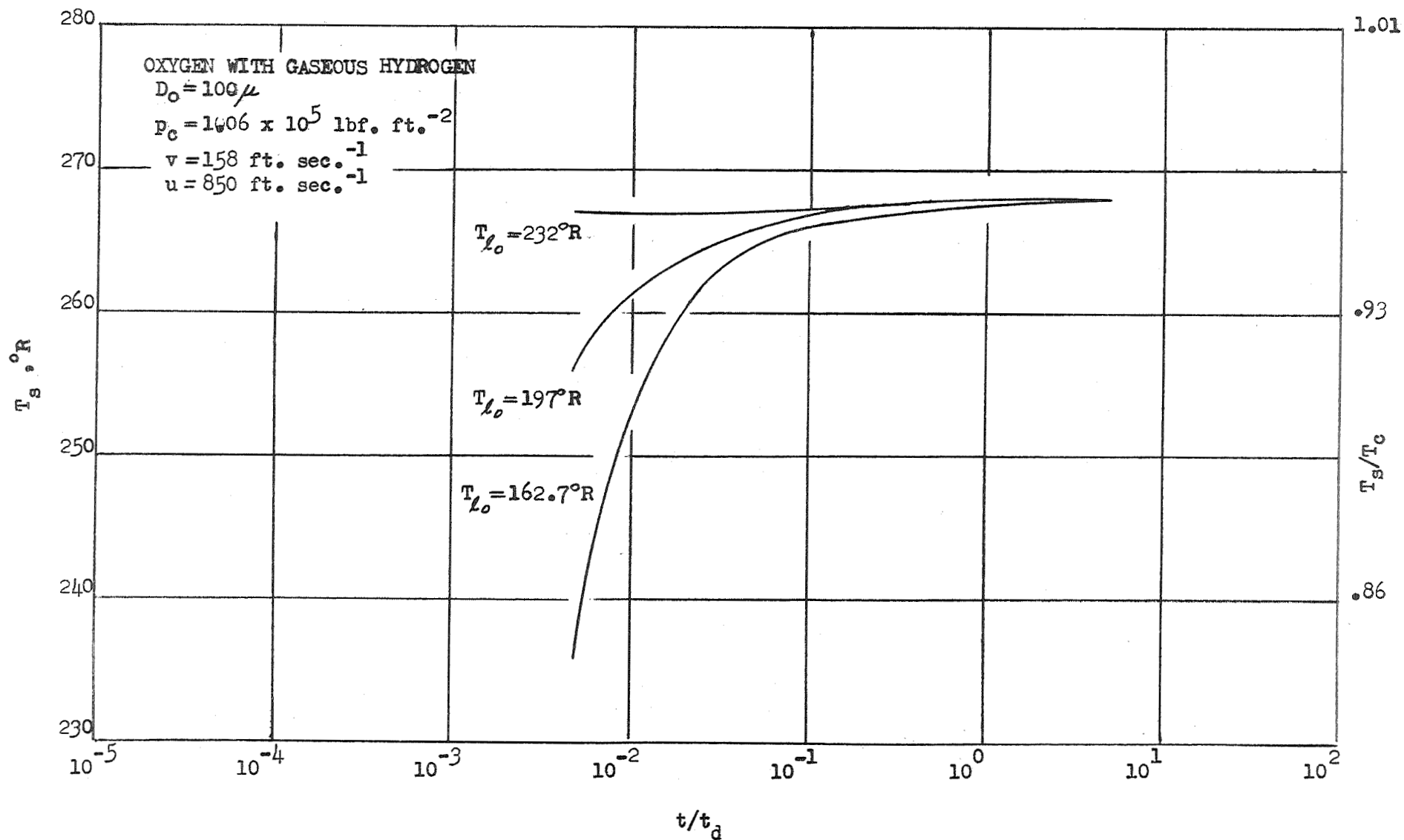


FIG. 6c. SURFACE TEMPERATURE VS. TIME FOR VARIATION IN INITIAL DROPLET TEMPERATURE, OXYGEN WITH GASEOUS HYDROGEN

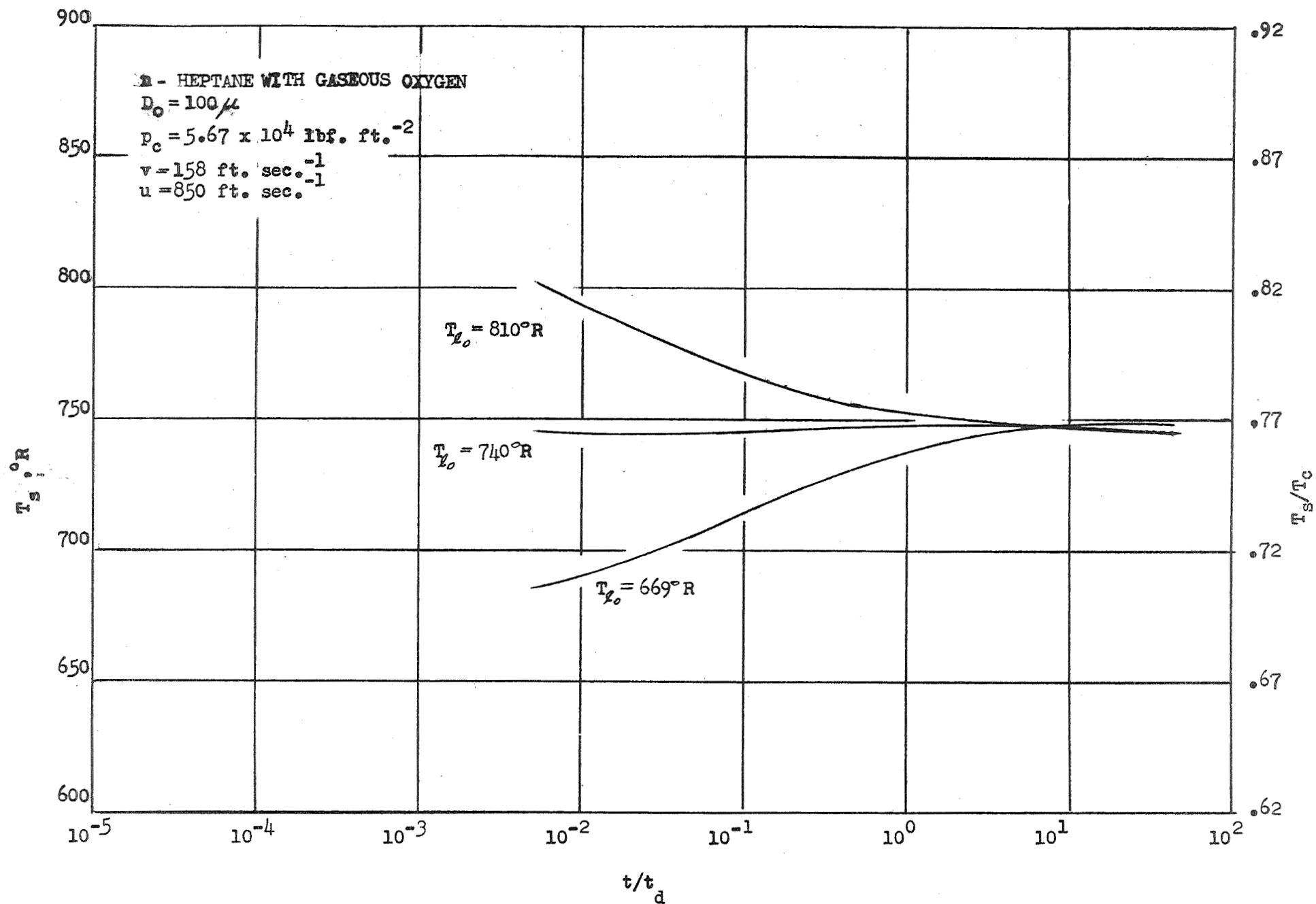


FIG. 6d. SURFACE TEMPERATURE VS. TIME FOR VARIATION IN INITIAL DROPLET TEMPERATURE, n-HEPTANE WITH GASEOUS OXYGEN



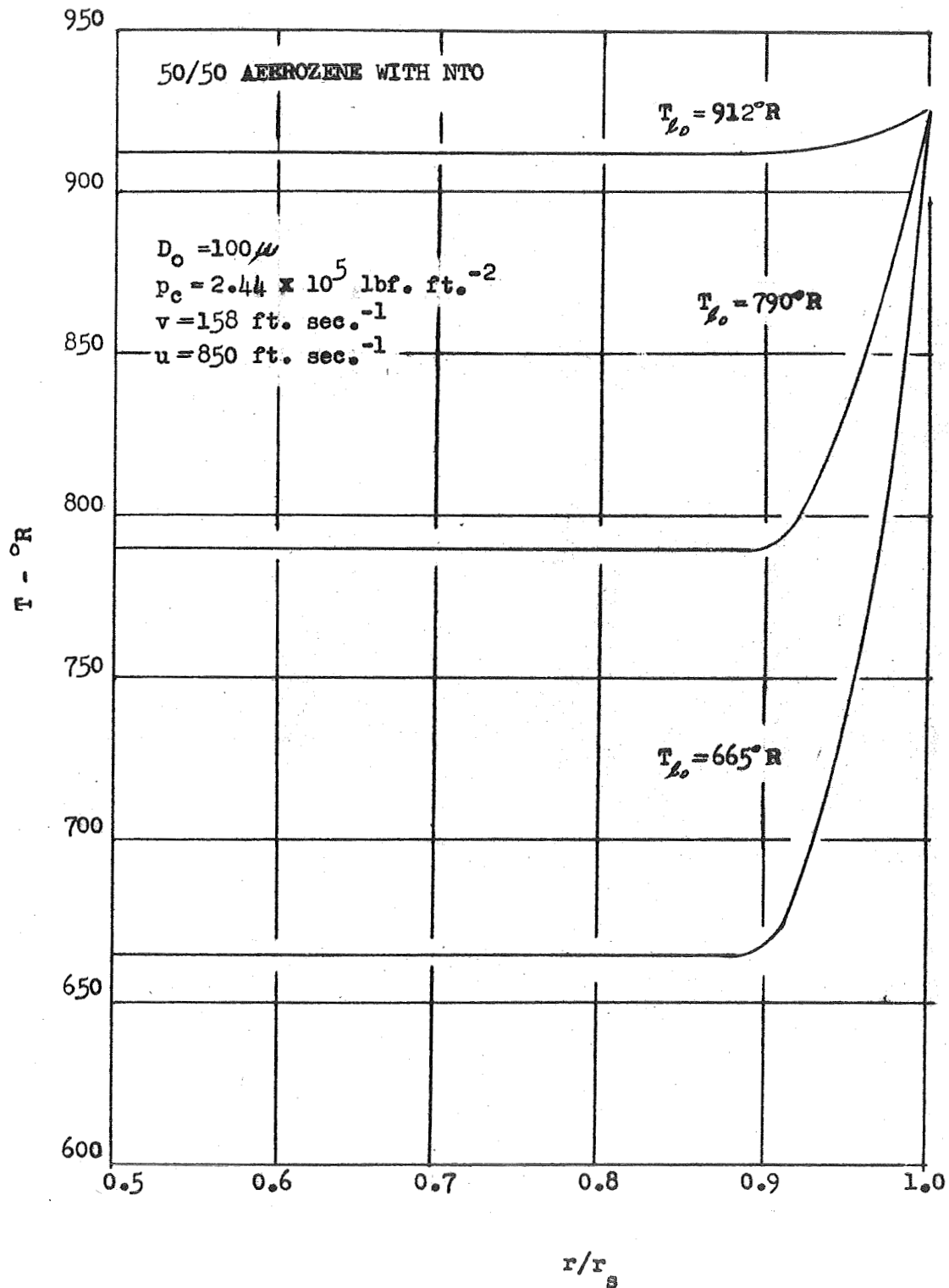


FIG. 7a. DROPLET TEMPERATURE DISTRIBUTION FOR VARIATION IN INITIAL DROPLET TEMPERATURE, AT A TIME OF  $10^{-4}$  SECOND, 50/50 AERROZENE WITH NTO

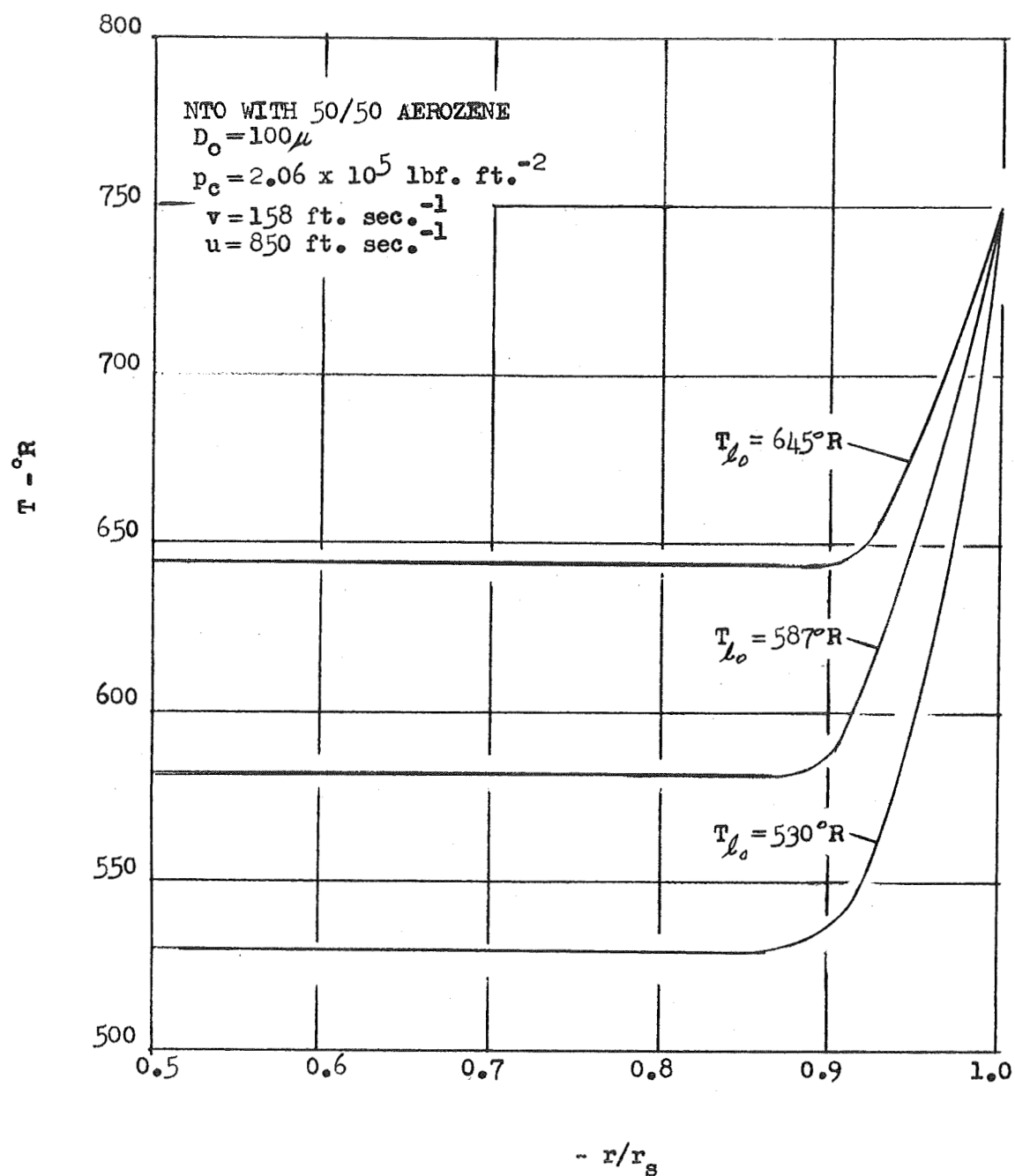


FIG. 7b. DROPLET TEMPERATURE DISTRIBUTION FOR VARIATION IN INITIAL DROPLET TEMPERATURE, AT A TIME OF  $10^{-4}$  SECOND, NTO WITH 50/50 AEROZINE

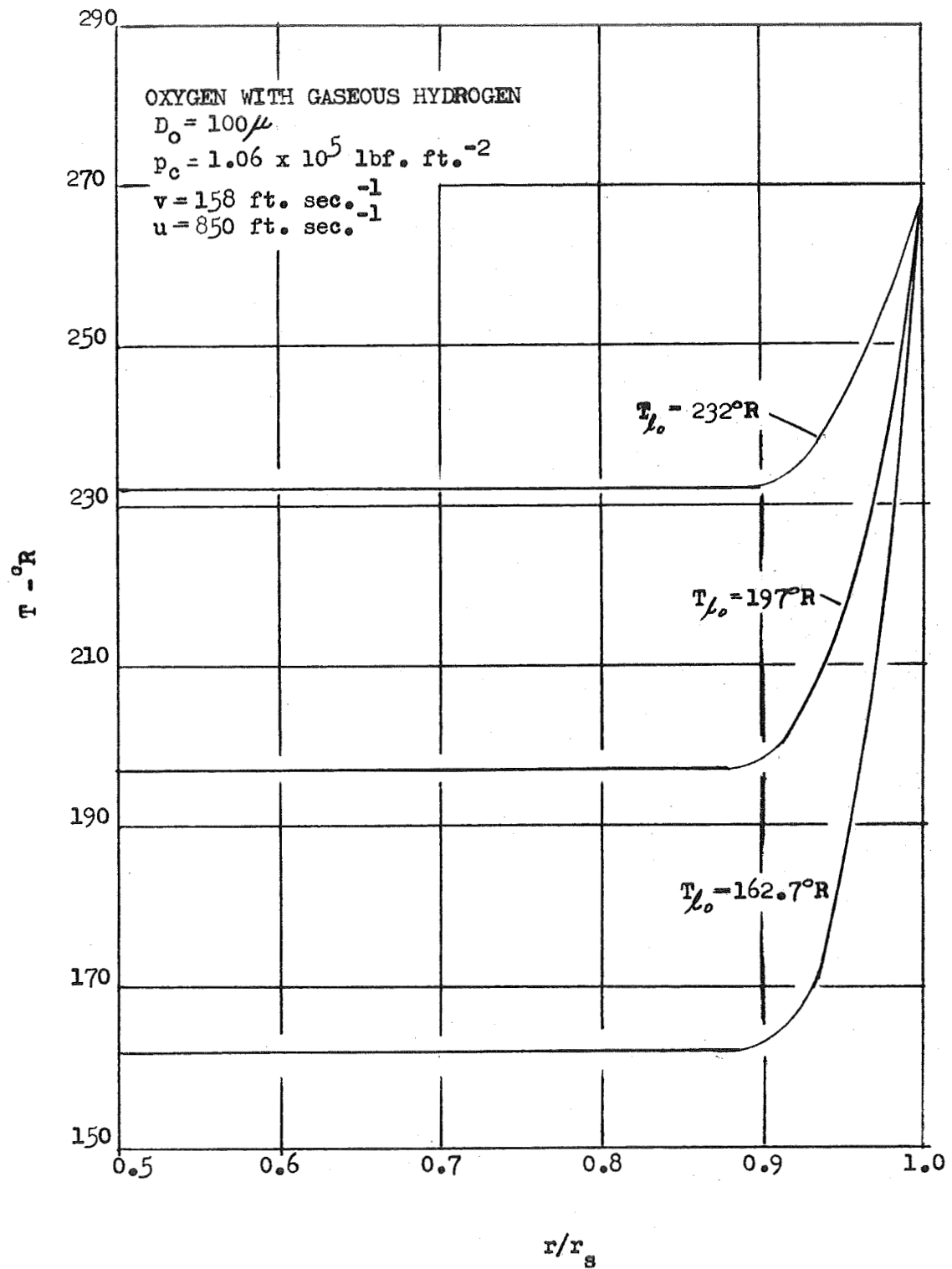


FIG. 7c. DROPLET TEMPERATURE DISTRIBUTION FOR VARIATION IN  
 INITIAL DROPLET TEMPERATURE, AT A TIME OF  $10^{-4}$  SECOND,  
 OXYGEN WITH GASEOUS HYDROGEN

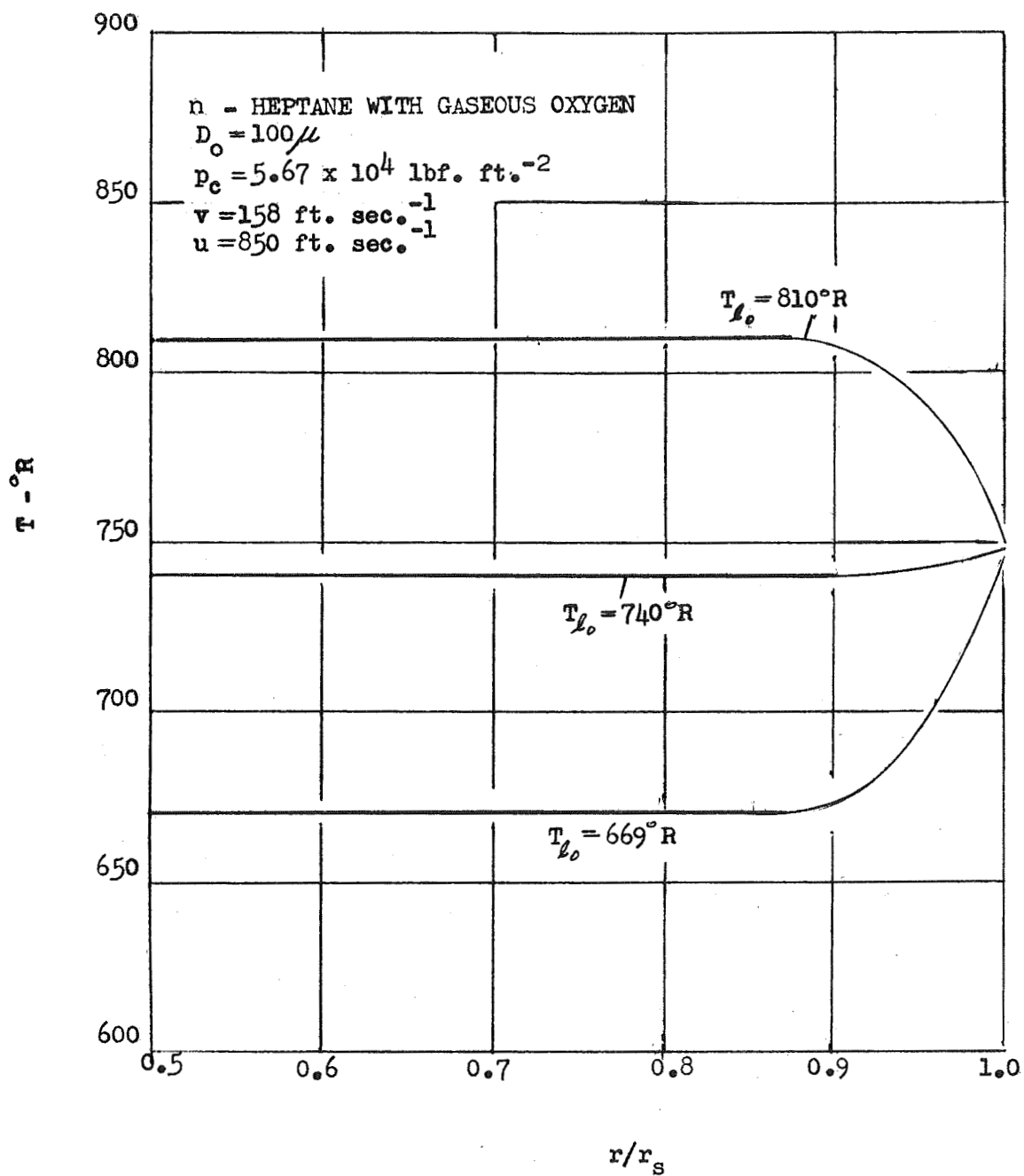


FIG. 7d. DROPLET TEMPERATURE DISTRIBUTION FOR VARIATION IN INITIAL DROPLET TEMPERATURE, AT A TIME OF  $10^{-4}$  SECOND, n-HEPTANE WITH GASEOUS OXYGEN

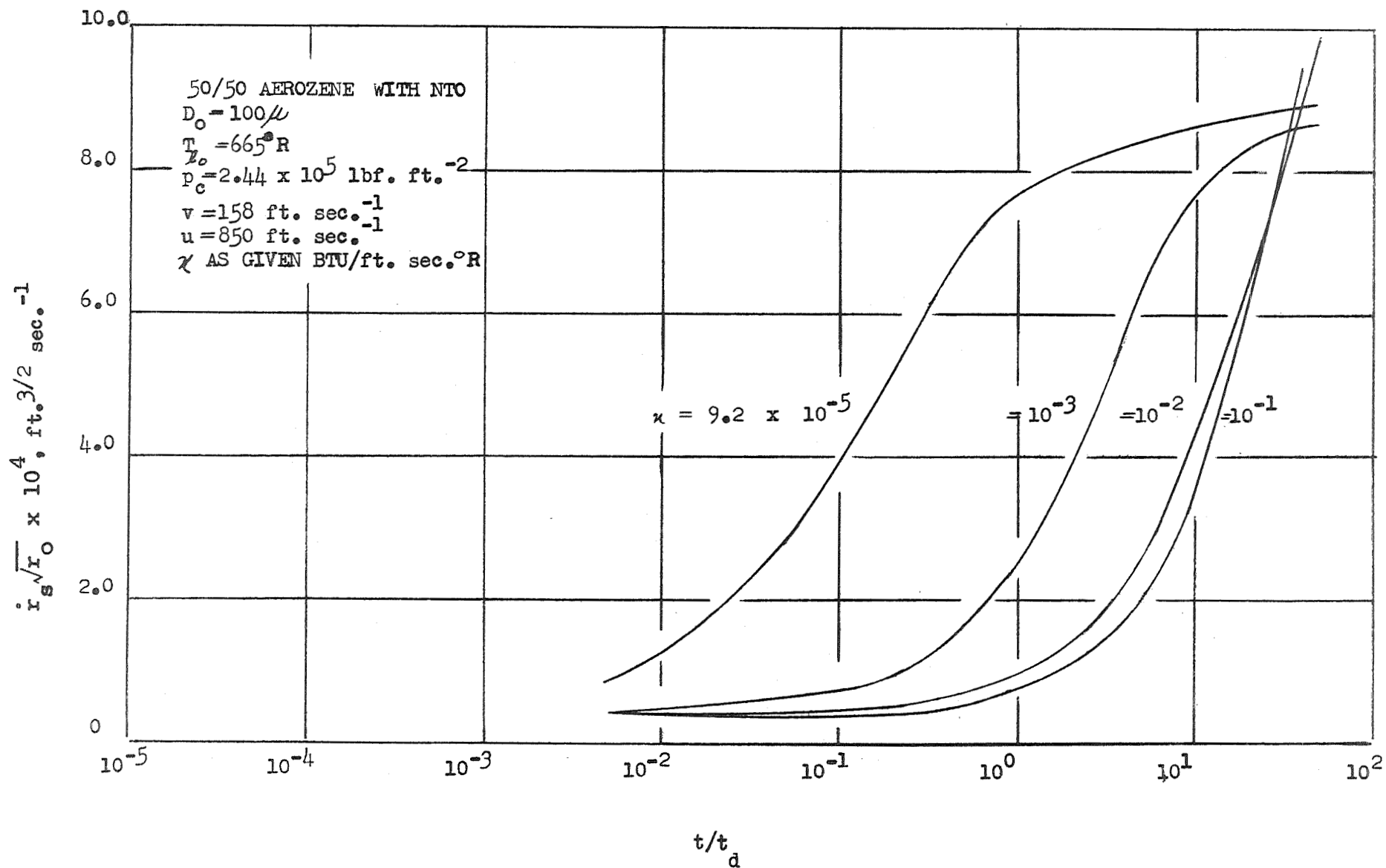


FIG. 8a. SURFACE REGRESSION RATE VS. TIME FOR VARIATION IN PROPELLANT THERMAL CONDUCTIVITY, 50/50 AEROZINE WITH NTO

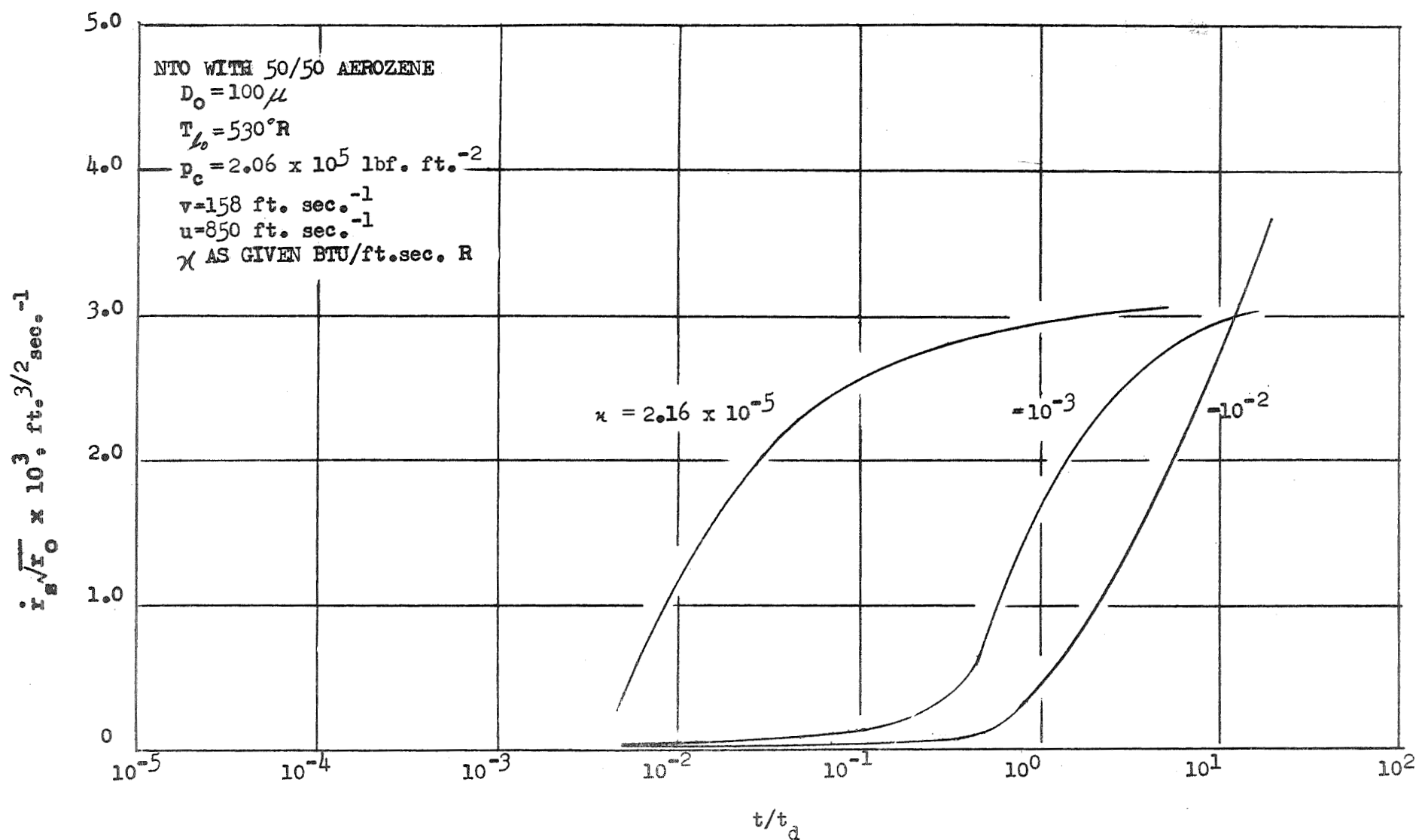


FIG. 8b. SURFACE REGRESSION RATE VS. TIME FOR VARIATION IN PROPELLANT THERMAL CONDUCTIVITY, NTO WITH 50/50 AEROZINE

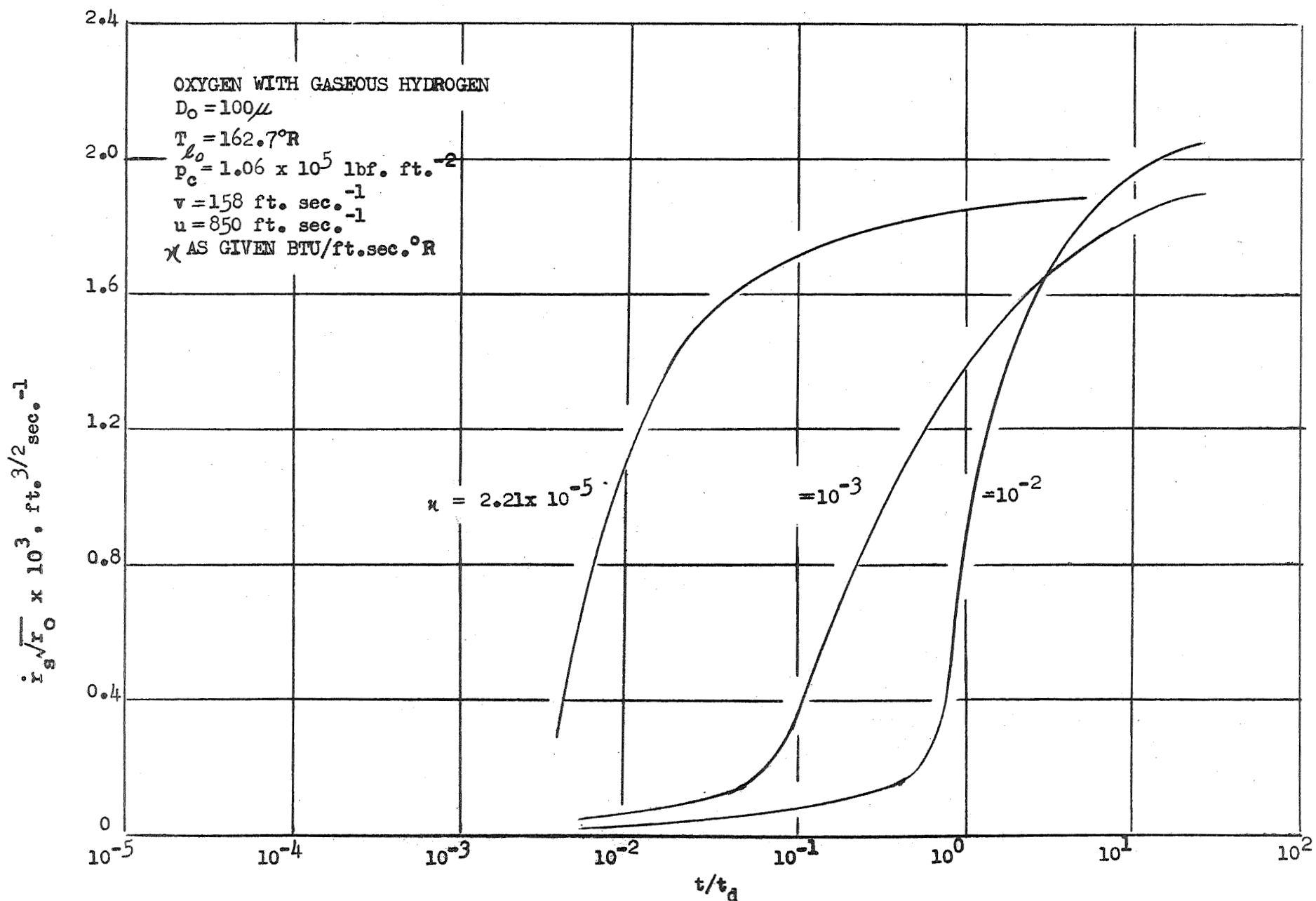


FIG. 8c. SURFACE REGRESSION RATE VS. TIME FOR VARIATION IN PROPELLANT THERMAL CONDUCTIVITY, OXYGEN WITH GASEOUS HYDROGEN

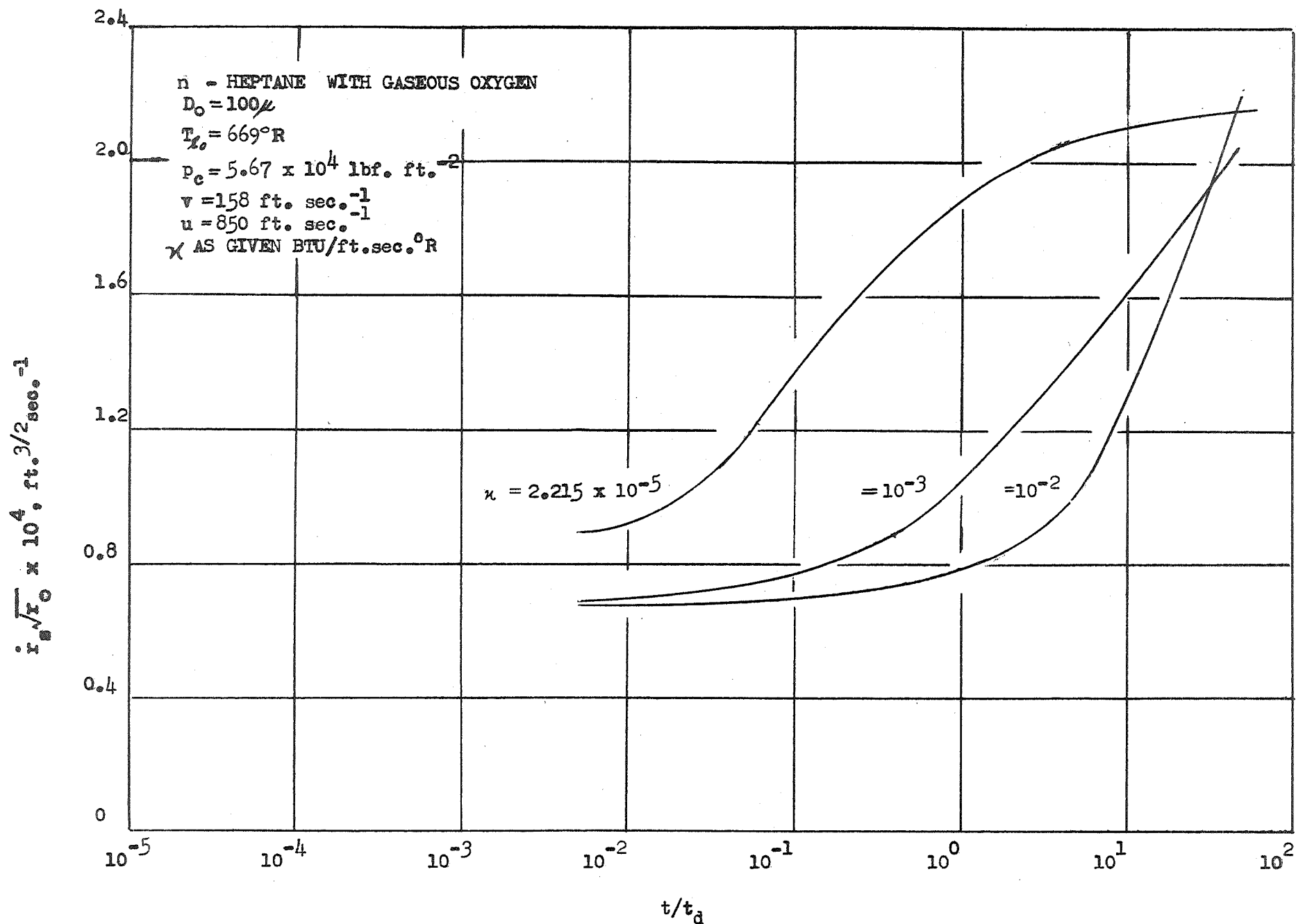


FIG. 8d. SURFACE REGRESSION RATE VS. TIME FOR VARIATION IN PROPELLANT THERMAL CONDUCTIVITY, n-HEPTANE WITH GASEOUS OXYGEN



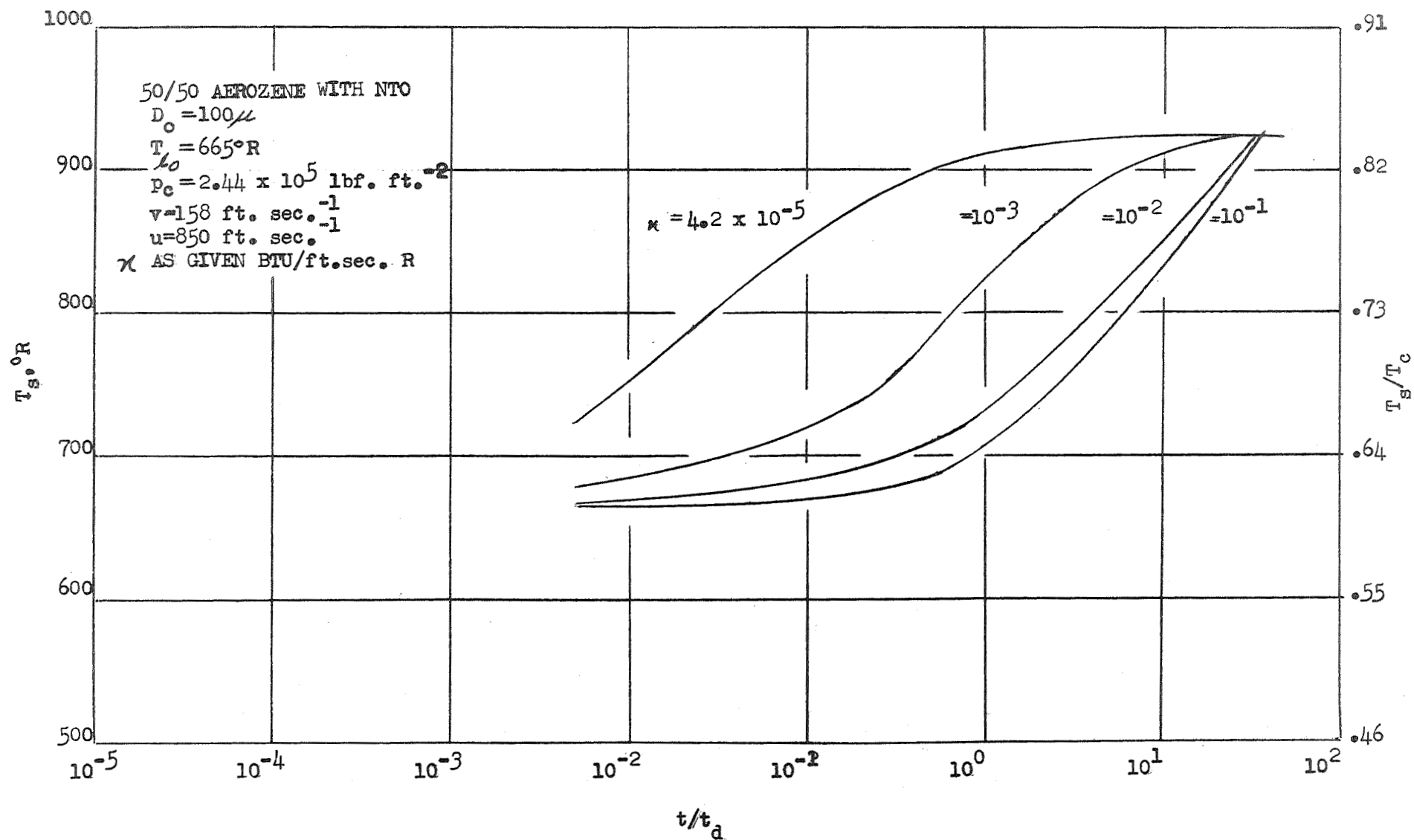


FIG. 9a. SURFACE TEMPERATURE VS. TIME, 50/50 AEROZINE WITH NTO

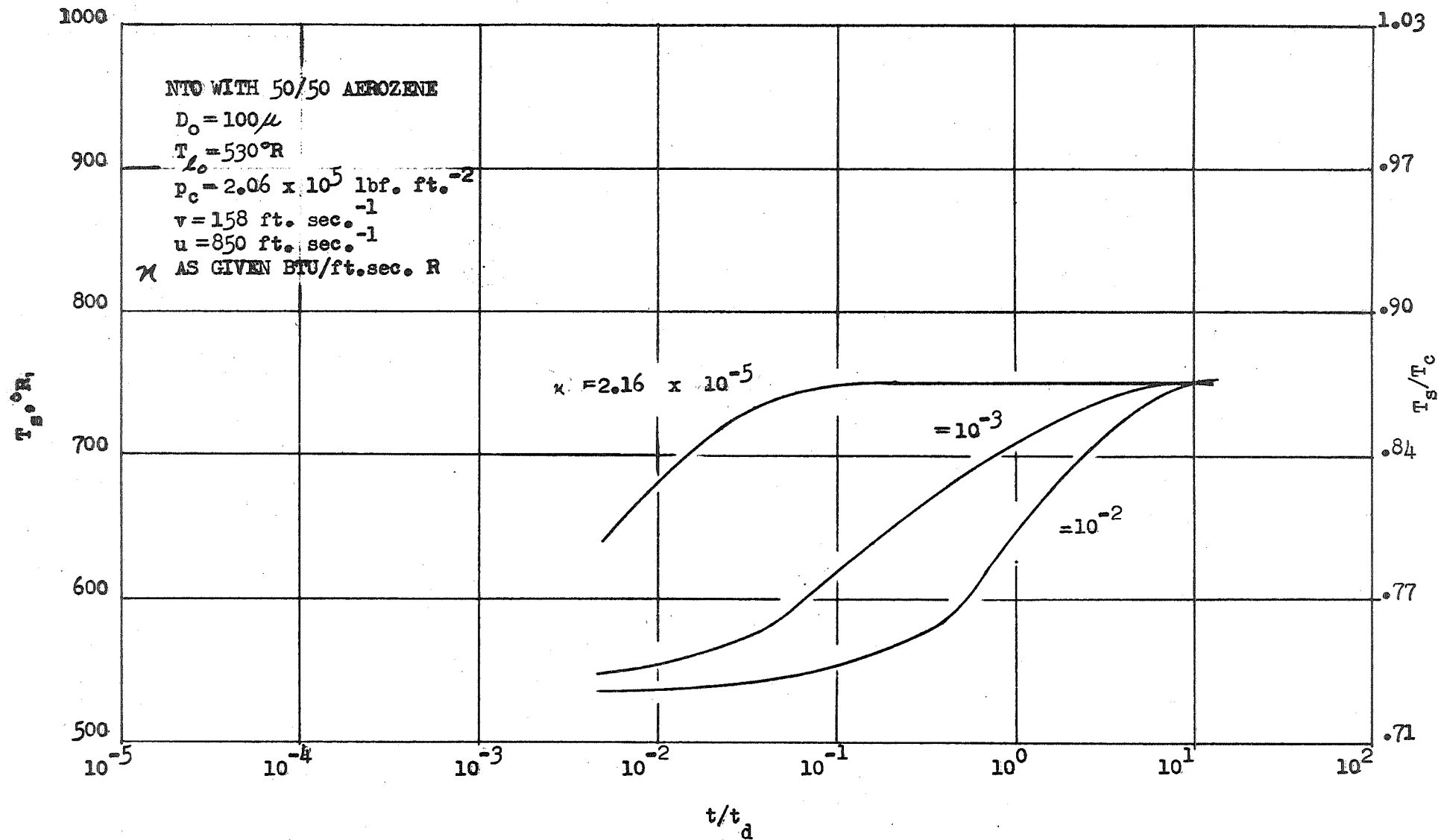


FIG. 9b. SURFACE TEMPERATURE VS. TIME, NTO WITH 50/50 AEROZINE

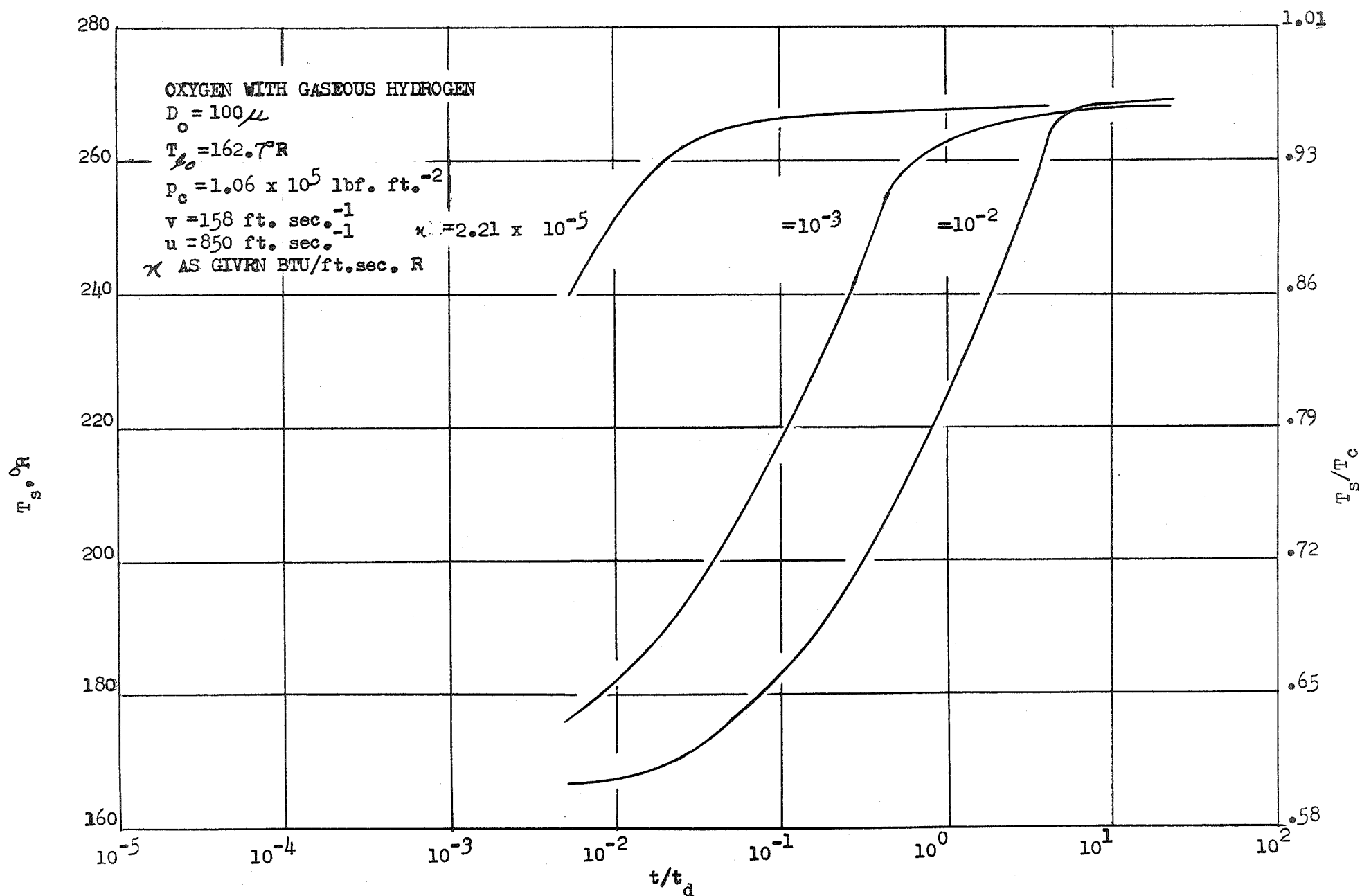


FIG. 9c. SURFACE TEMPERATURE VS. TIME, OXYGEN WITH GASEOUS HYDROGEN

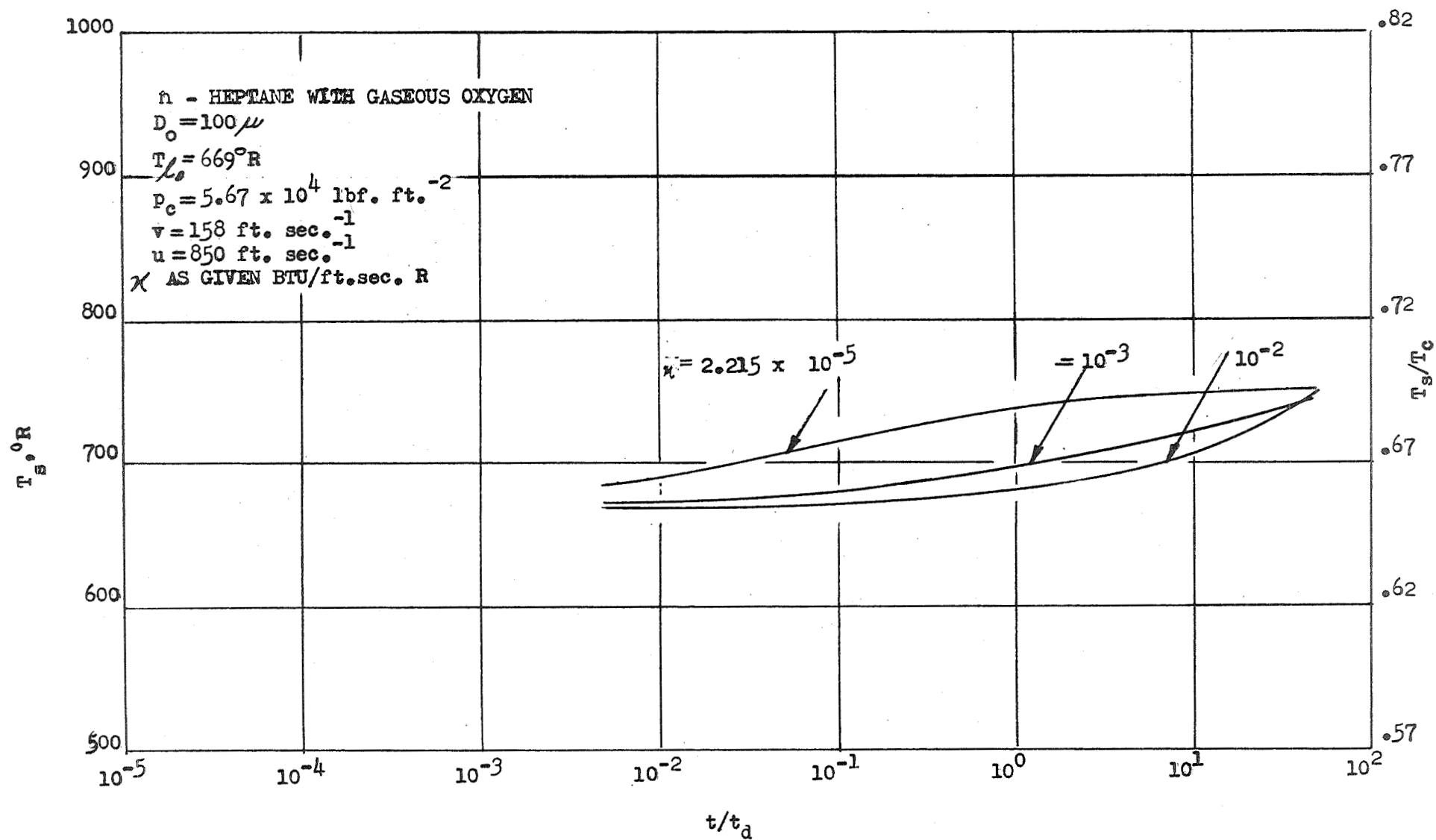


FIG. 9d. SURFACE TEMPERATURE VS. TIME, n-HEPTANE WITH GASEOUS OXYGEN

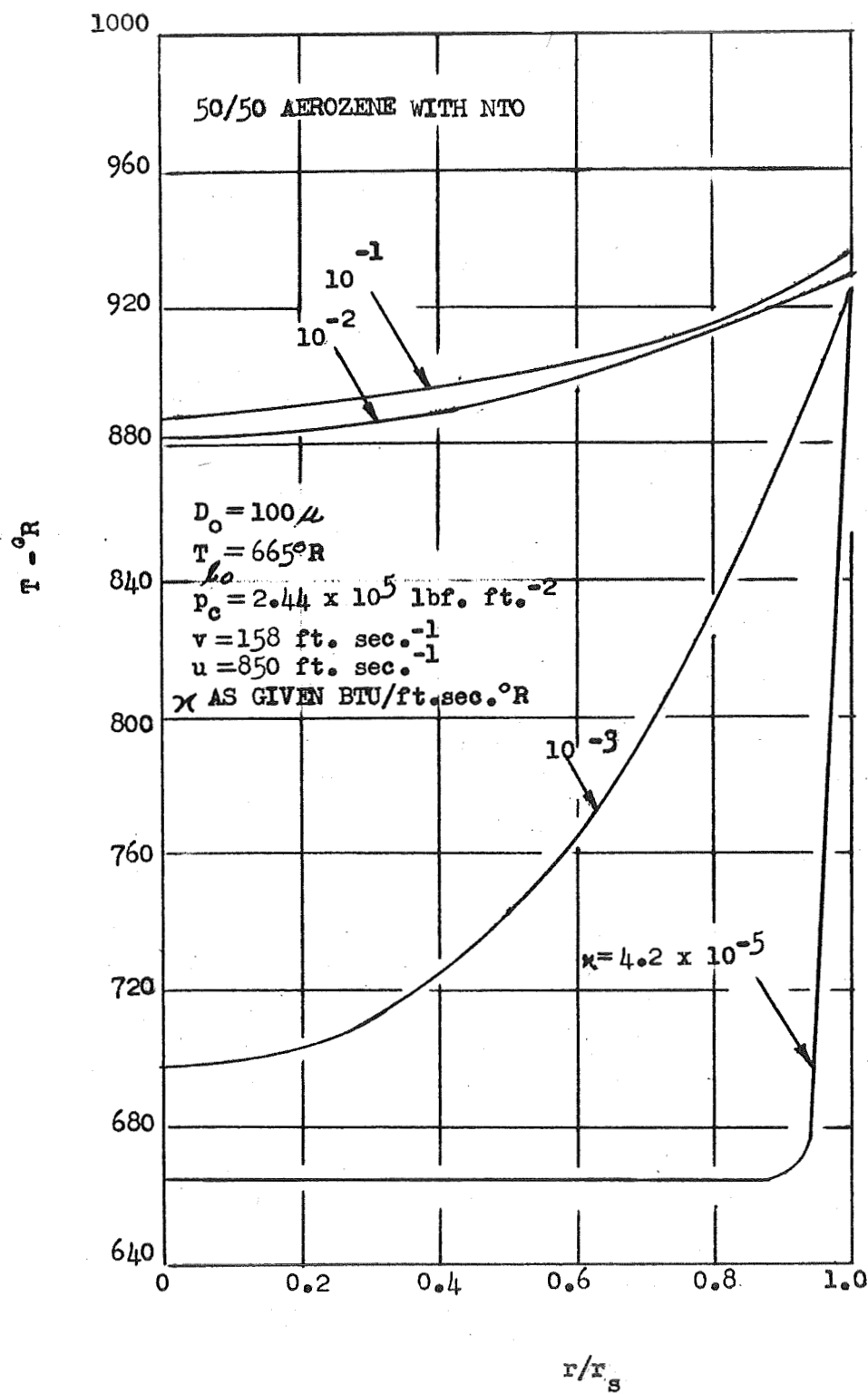


FIG. 10a. DROPLET TEMPERATURE DISTRIBUTION, AT TEMPERCENT DROPLET MASS EVAPORATION, 50/50 AEROSINE WITH NTO

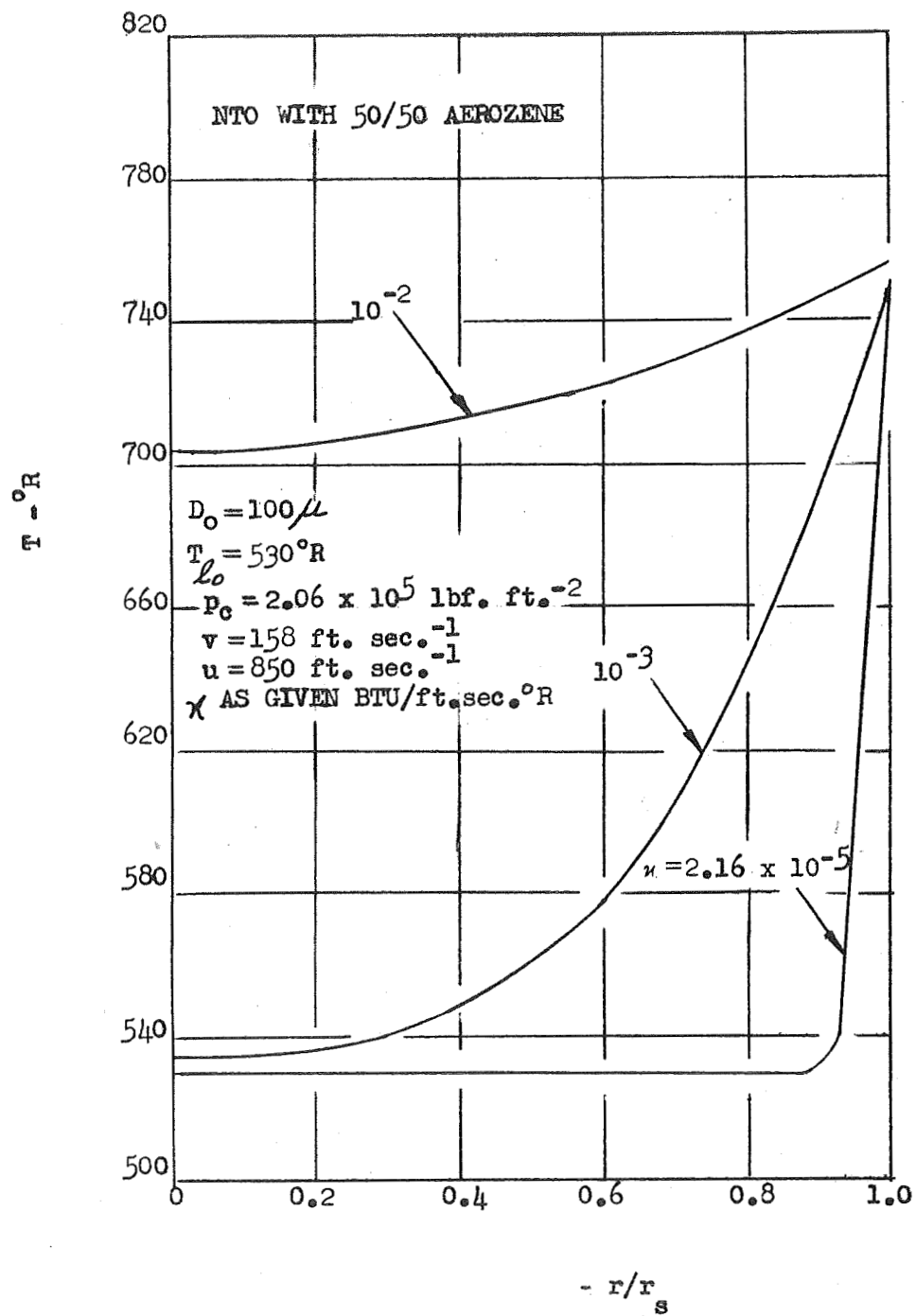


FIG. 10b. DROPLET TEMPERATURE DISTRIBUTION, AT TEN PERCENT DROPLET MASS EVAPORATION, NTO WITH 50/50 AEROZINE

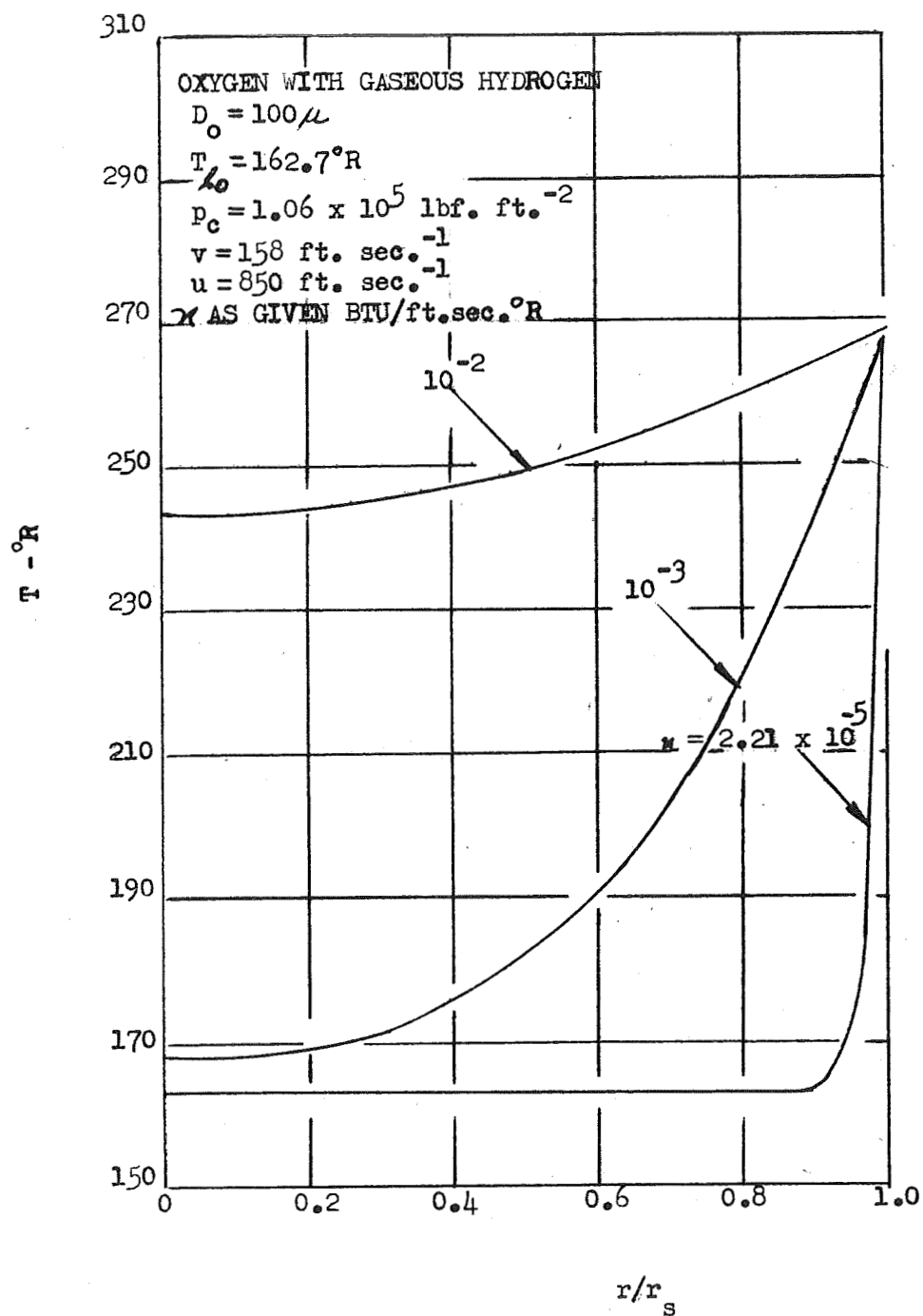


FIG. 10c. DROPLET TEMPERATURE DISTRIBUTION, AT TEN PERCENT DROPLET MASS EVAPORATION, OXYGEN WITH GASEOUS HYDROGEN

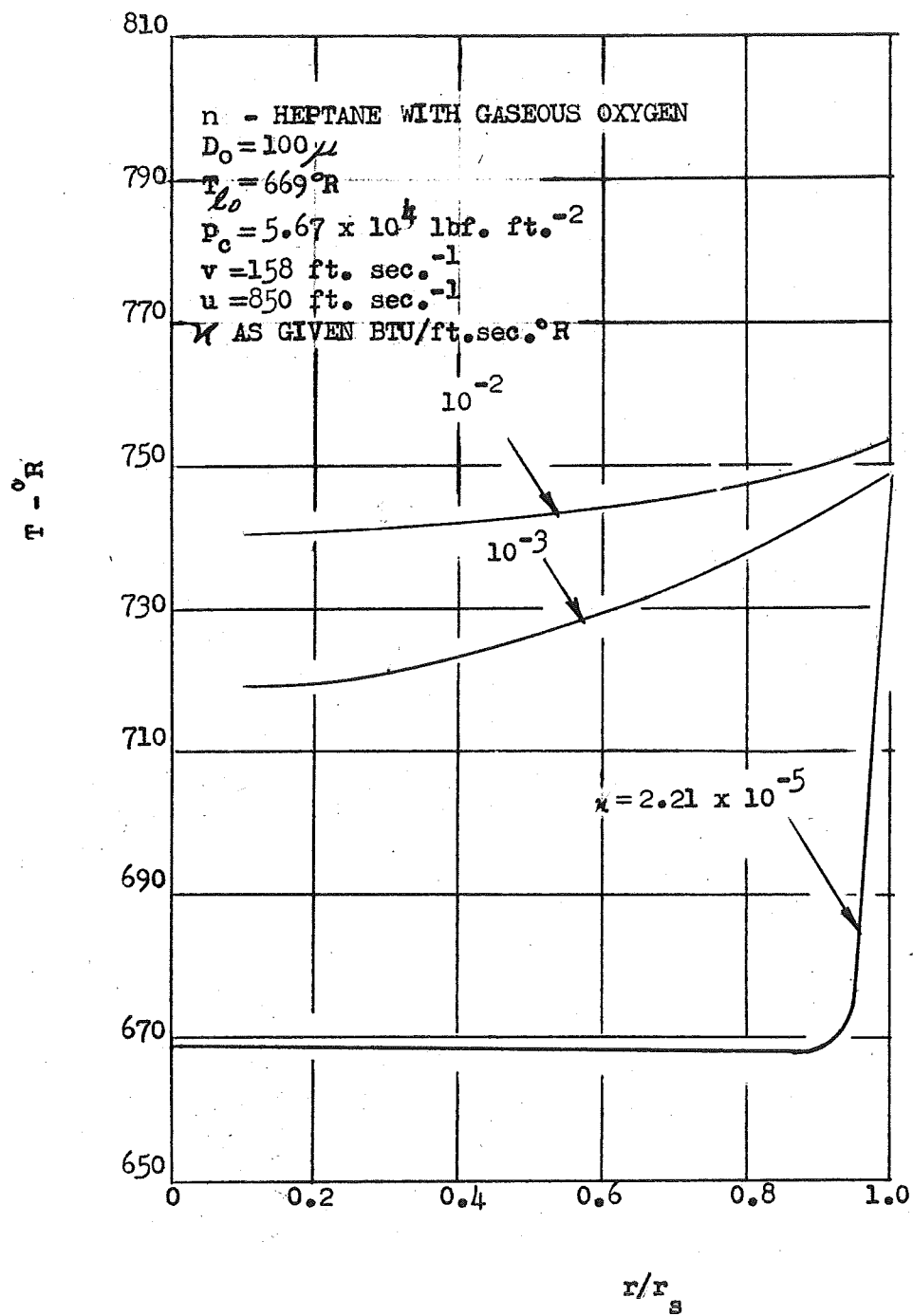


FIG. 10d. DROPLET TEMPERATURE DISTRIBUTION, AT TEN PERCENT DROPLET MASS EVAPORATION, n-HEPTANE WITH GASEOUS OXYGEN



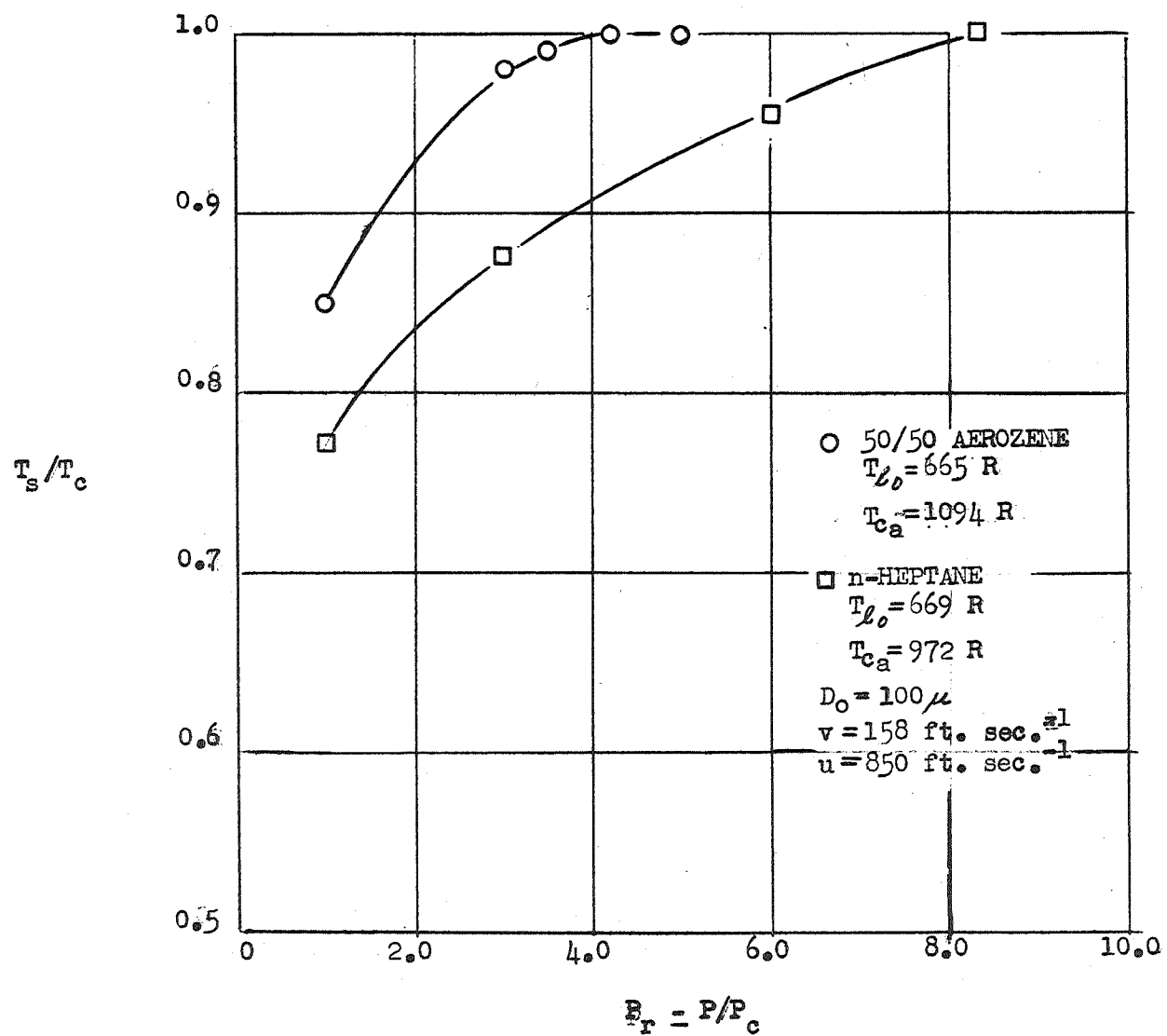


FIG. 11a. DROPLET SURFACE TEMPERATURE VS. ENVIRONMENTAL GAS PRESSURE, n-HEPTANE WITH GASEOUS OXYGEN, 50/50 AEROZINE WITH NTO

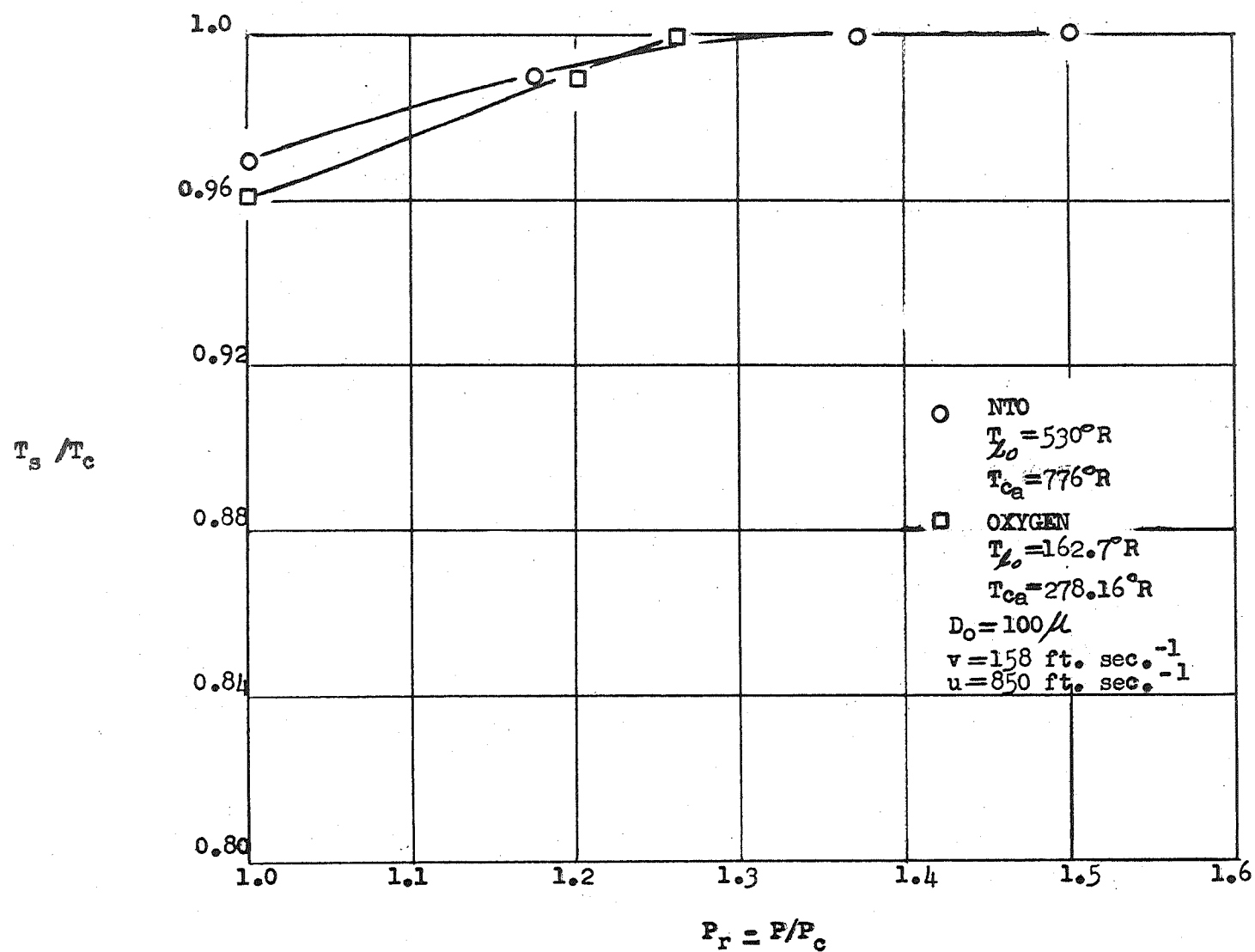


FIG. 11b. DROPLET SURFACE TEMPERATURE VS. ENVIRONMENTAL GAS PRESSURE, NTO WITH 50/50 AEROZINE, OXYGEN WITH GASEOUS HYDROGEN

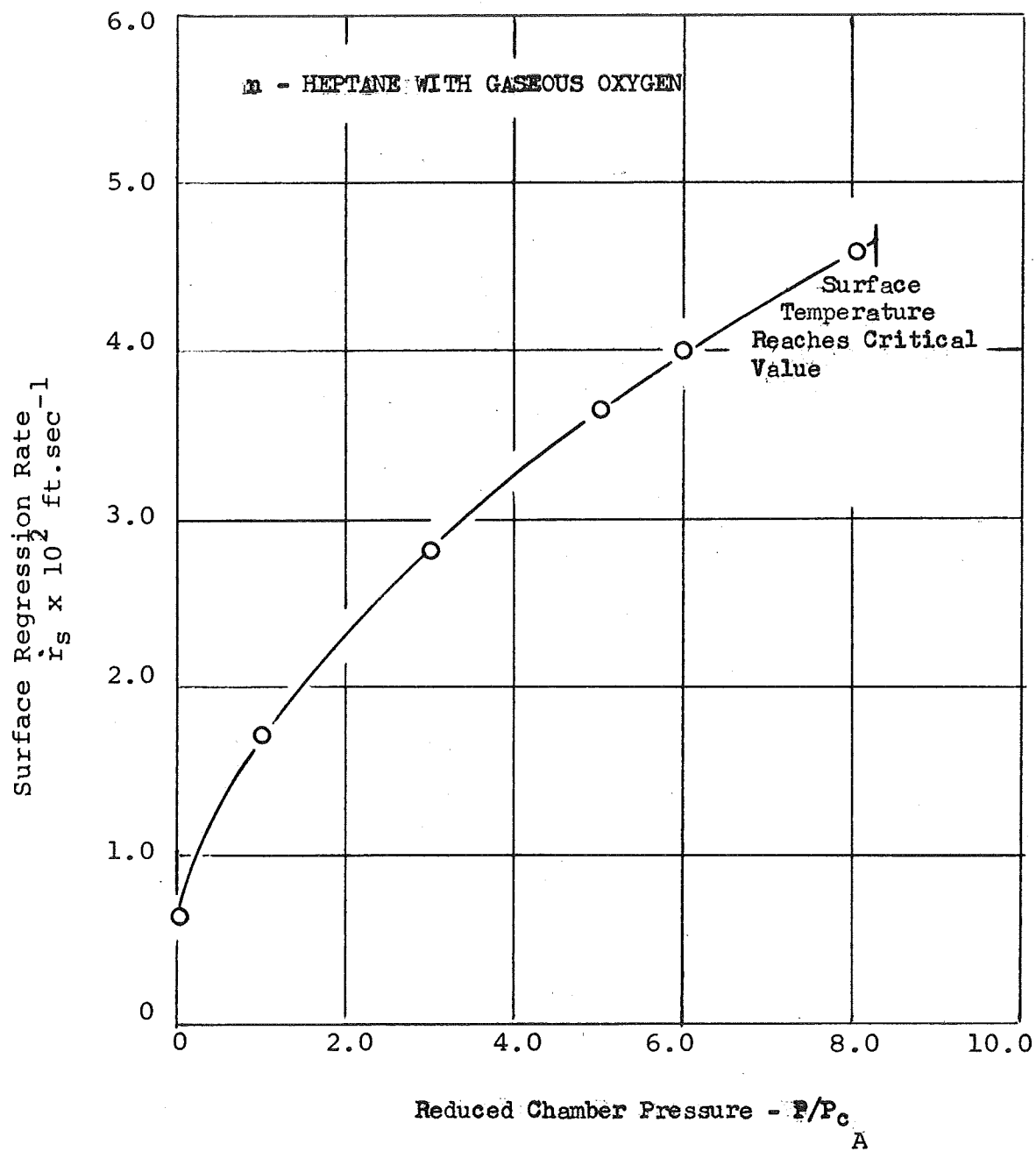


FIG. 12 SURFACE REGRESSION RATE AT SUPERCRITICAL PRESSURES FOR n-HEPTANE

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